

## Chapter 9

# Choice of the Structure of the Energy System of Complex Buildings in the Course of Preliminary Design

Traditional methods for choosing the structure of the energy management of complex buildings are based on the heuristic knowledge of designers, and as such they are restricted only to analyses of a few variants. Nowadays, this is insufficient due to the permanently growing amount of technical and economic information about new techniques in the production of electricity, heat, and cooling agents. This is in so far important that errors in the choice of the structure may cause not only unjustifiably high expenditures of investment but also higher expenditures of exploitation.

In order to avoid these errors, systems approach should be applied permitting to analyze all the possible justifiable variants of energy management of complex buildings. In the systems, approach to choosing the optimal structure of the energy management of complex buildings the hierarchical feature of large energy systems and the input–output analysis should be applied.

As in the case of the traditional design method, the first stage ought to be the preparation of the scenario for the predicated energy management of a concrete complex buildings, taking into account its specific characteristics (in a supermarket, for instance, only hot water is used as a heat carrier, whereas in a hospital also steam is required as a high-temperature heat carrier). Such a scenario serves to set up a general specification of energy carriers comprising both energy carriers produced in complex buildings and those supplied from outside (e.g., fuels and raw water). Besides this general specification, a list of energy facilities which may be applied in the designed energy management of complex buildings should also be set up based on this scenario.

This then serves to form the matrix of the projects and subsets of designs, from which the variants of energy management of complex buildings are derived. To each variant corresponds a calculation diagram of energy management and a binary input–output matrix. Later on, this matrix is subjected to structural analysis resulting finally in a matrix with a minimum number of nonzero elements below

the main diagonal. In this way the number of feedback interconnections is reduced to a minimum.

The mathematical optimization model for choosing the structure of energy management of complex buildings is based on the decomposition of the global optimization task. Lagrange's method of decomposition is applied. It has been proved that the coordination procedure is a matrix method of calculating the unit costs of energy carriers.

The descriptive model of this system method has been applied in the case of a typical office building, and the optimization model was applied in the choice of the optimal energy management structure of an industrial-urban complex. In large urban districts energy subsystems may frequently serve both industrial consumers and complex buildings (e.g., sports and recreation centers).

## **9.1 Systems Approach to the Preliminary Design of the Energy Management of Complex Buildings**

In designing the following three phases may be distinguished:

- feasibility study of the design for the purpose of choosing a subset of useful solutions from a set of probably correct solutions, taking into account the possibility of realizing them technically and the financial possibilities,
- preliminary design consisting in the choice of the optimal variant,
- detailed design aimed at providing the technical description of the optimal variant of the designed energy management system of the complex buildings.

In the course of preliminary design the optimal structure of energy management for complex buildings is chosen. The energy management structure is to be understood as a set of the main energy installations and engines determined by the power ratings and their number, as well as the configurations of their interconnections [3, 4]. The set of possible variants of the energy management structure of complex buildings is abundant due to the large variety of elements (energy installations and engines) constituting the energy subsystem, numerous variants of thermodynamic parameters characterizing these elements, as well as numerous possible combinations of interconnections of these elements. The task of the designer is to find those variants, to check the technical possibilities of their realization, assessing each of them from the economic point of view, and by comparing them to choose the optimal one. This is the aim of the preliminary design. The dimension of the task of choosing the optimal structure grows much more, if the uncertainty of the input data is taken into account. Therefore, in the phase of preliminary design a systems analysis is indispensable.

The choice of the energy management structure of complex buildings plays an important role in the process of designing due to the fact that the building sector is one of the main consumers of energy [2, 9, 13]. The quality of final design depends

on the technical solutions (kind of technology and equipment) assumed in that stage. Errors in the choice of the structure greatly affect the excess of capital investments and also the incorrect functioning of the designed energy subsystem. Traditionally, the most often applied method for choosing the energy management structure comprises the elaboration of several variants and the choice of the best one from among them. This is a heuristic approach based on hitherto gathered experience of designers. Nowadays, this traditional approach has proven to be insufficient due to the continuous development of technology requiring new solutions, as well as the permanently growing amount of technical-economic information which ought to be utilized in designing [11].

The choice of the energy management structure of complex buildings should be based on a systematic review of all admissible structures. This requires the application of systems analysis [8]. The systems approach is to be understood as a way of solving problems in which the processes are treated to be complex from the point of view of internal and external interdependences [14].

Up-to-date designing applying systems analysis, comprises the following modules [7]:

- introduction of design data,
- search for information,
- mathematical modeling and optimization,
- software,
- automatic preparation of documentation.

The module of the introduction of data permits the computer store to be provided with information concerning the designed energy subsystem (e.g., calculation diagram of the energy management system). The module of searching for information contains, for instance, a series of types of machines and other equipment to form the data base.

Modeling and optimization comprise algorithmic and functional modules. The former is a procedure for calculating the designed element of the energy management system of complex buildings. Functional modules constitute organizational units of the subsystem of modeling and optimization, by means of which new variants of the energy management system can be obtained. The algorithmic module is formulated in compliance with principles obligatory in system engineering and consist of the following stages:

- formulation of the problem,
- elaboration of a descriptive and optimizing mathematical model,
- solution by means of the model,
- verification of the model and the obtained solution,
- control of the solution,
- practical application of the solution.

The module of modeling and optimization allows the technique of simulation, to be practically applied, providing the following advantages:

- the possibility of checking a larger number of variables than in direct investigations,
- lower costs and a shorter duration of investigations,
- the possibility of attaining and checking conditions which do not yet exist in reality.

The structure of the energy subsystem presented as a calculation diagram, oriented graph, or binary matrix of interbranch connections illustrates how the engines and energy installations are connected without presenting their physical form. This serves to describe the energy subsystem qualitatively. Therefore, it is often called a topological structure.

The topological structure is determined in the course of preliminary designing concerning each variant of the energy subsystem analyzed by the designer. Further on the quantitative characteristics are determined for both the systems of machines and energy equipment in the respective branches and their interbranch connections. These are first of all the power rating and capacity of the engines and energy installations and their number, as well as the coefficients of consumption and by-production of energy carriers. In this stage of preliminary design for each variant of the energy subsystem, the specifications of the systems of machines and equipment are determined. Next, the optimal technical structure is chosen.

The choice of the structure of the energy subsystem, being the aim of the preliminary design, is therefore the optimal task. As the task of optimization is a complex one, due to the variety of algorithms of the choice of equipment for the respective energy carriers, the global optimization task must be decomposed [10]. For this purpose the hierarchical feature of the energy subsystem is helpful. When optimizing the respective branches, the power ratings of engines and energy installations are determined and also the nominal flux of external supplies of energy carriers (mainly fuels). Therefore, knowledge of the duration curves for each energy carrier and the technical coefficients of consumption and by-production of energy carriers is indispensable. These coefficients depend both on the value of the power rating and the load. Thus, plotting the duration curves, the determination of the coefficients of consumption and by-production of energy carriers, and the proper choice of power ratings, are interdependent tasks, solved iteratively [17].

## **9.2 Choice of the Variants of Energy Management of Complex Buildings**

The procedure for choosing the adequate structure of the energy management of complex buildings has been worked out based on the input–output model concerning the interbranch flow of energy carriers. The choice of this structure in the course of preliminary design consists in the formation of an optimal set of energy machines and equipment and the determination of interconnections between them resulting from the flows of energy carriers. First of all available techniques of

**Table 9.1** General list of energy carriers and the structure of the vectors  $\mathbf{B}_G^b$  and  $\mathbf{B}_D^b$  production and supplies of energy carriers and vector  $\mathbf{O}^b$  of the demand for energy carriers for the subsystem of consumers

$i$	Energy carrier	$\mathbf{B}_G^b$	$\mathbf{B}_D^b$	$\mathbf{O}^b$
1	Electricity	1	0	1
2	Heat	1	0	0
3	Cold	1	0	0
4	Hot process water for absorption chiller	1	0	0
5	Cooling medium 6/12 °C	1	0	1
6	Hot process water I 60/45 °C	1	0	1
7	Hot process water II 85/55 °C	1	0	1
8	Hot tap water	1	0	1
9	Air from air conditioning unit	1	0	1
10	Ventilation air	1	0	1
11	Drinking water	0	1	1
12	Natural gas	0	1	1
13	Heating oil	0	1	0
14	Diesel oil	0	1	0

producing energy carriers must be chosen, based on which the scenarios of energy management are developed for the given subsystem of energy consumers. The scenario is a verbal description of the method for realizing the energy management concerning the complex buildings under consideration, characterized by the vector of the demand for energy carriers covering the need of the subsystem of consumers.

The next step is to set up a universal specification of energy carriers comprising the main production of energy carriers, the by-production, and energy supplies from outside. This universal specification of energy carriers contains energy carriers, the production of which has to cover the demand of the subsystem of consumers, taking also into account energy carriers produced as a result of interbranch connections in the energy subsystem. The energy carriers quoted in the universal specification may be divided into two groups [15]:

- energy carriers produced in the energy subsystem as main products,
- external energy supplies and energy carriers being by-products in the energy subsystem which do not supplement the main production and the supplementary external supplies.

The energy carriers in this universal specification are described by binary vectors defining whether the given energy carrier is a main product (vector  $\mathbf{B}_G^b$ ) or is supplied from outside (vector  $\mathbf{B}_D^b$ ) and whether it is consumed directly in the subsystem of consumers (vector  $\mathbf{O}^b$ ) (Table 9.1 as an example cf. 9.7).

Besides the universal specification of energy carriers consumed in complex buildings a data base is set up concerning the thermal, electrical, and cooling techniques applied in these complex buildings (Table 9.2 as an example cf. 9.7).

**Table 9.2** Energy equipment for the considered office building

Symbol	Equipment
U1	Refrigeration unit
U2	Absorption chiller I—powered by hot water
U3	Absorption chiller II—powered by hot natural gas
U4	Water heaters powered by heating oil
U5	Water heaters powered by natural gas
U6	CHP system with piston engine powered by diesel oil
U7	CHP system with microturbine powered by natural gas
U8	CHP system with fuel cell powered by natural gas
U9	Peak water heater powered by natural gas
U10	Peak water heater powered by heating oil
U11	Peak refrigeration unit
U12	Air conditioning unit
U13	Exhaust ventilation system
U14	Heat exchanger—cooling medium 6/12 °C
U15	Heat exchanger—hot water 60/45 °C
U16	Heat exchanger—hot water 85/55 °C
U17	Heat exchanger—hot tap water

This data base serves to determine the technical coefficients of the input–output matrix characteristic for the table of interbranch flows.

At this stage, the structure of the energy subsystem may be presented in the form of a calculation diagram and a corresponding oriented graph. The structure of the energy subsystem can also be described by the binary matrix of interbranch connections [16, 23]. The binary matrix is used to set up a table of interbranch connections of energy carriers in complex buildings. This table serves as the basis for the mathematical model of the energy management balance of the complex buildings.

Each energy carrier which is a main product corresponds to the design topic, which is connected with a subset of technical solutions. A technical solution is to be understood as a set of engines and energy installations realizing the production of the given energy carrier. One design topic corresponds to several technical solutions. The production of heat in complex buildings, for instance, can be realized in a cogeneration unit, heating water boilers, or a modern fuel cell. The set of design topics and technical solutions can be presented best as a binary matrix  $\Pi$  (Table 9.3 as an example cf. 9.7). This matrix comprises all the design topics and technical solutions which predict certain energy equipment or the possibility of providing energy carriers from outside. The respective rows in the matrix correspond to a given technical solution and the columns to a given engine or energy installation.

Every technical solution is described by binary matrices of the connections  $\mathbf{A}_p^b = [a_{p\ ij}^b]$  and  $\mathbf{F}_p^b = [f_{p\ ij}^b]$ , the elements of which are defined as follows:

$a_{p\ ij}^b = 1$  if the  $i$ th energy carrier is consumed in the  $p$ th technical solution for the production of the  $j$ th energy carrier,

Table 9.3 Matrix of the projects and subsets of design Π

Project	No. of design	U1	U2	U3	U4	U5	U6	U7	U8	U9	U10	U11	U12	U13	U14	U15	U16	U17	SUPPLY
1 Electricity	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	2	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1
	3	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
	4	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
2 Heat	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	6	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
	8	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	9	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
3 Cold	15	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
	16	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	17	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
	18	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
4 Hot process water for absorption chiller I	19	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	20	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	21	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	22	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
5 Cooling medium I 6/12°C	23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
6 Hot process water I 60/45°C	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
7 Hot process water II 85/55°C	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
8 Hot tap water	26	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
9 Air from air conditioning unit	27	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
10 Ventilation air		0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0

- $a_{pij}^b = 0$     the opposite of the former one,  
 $f_{pij}^b = 1$     if the  $i$ th energy carrier is a by-product in the  $p$ th technical solution for the production of the  $j$ th energy carrier,  
 $f_{pij}^b = 0$     the opposite of the former one.

where

$$p = 1, 2, \dots, m$$

$$i = 1, 2, \dots, n$$

$$j = 1, 2, \dots, n_g$$

- $m$     number of technical solutions,  
 $n$     number of energy carriers in the universal specification,  
 $n_g$     number of energy carriers produced as main products.

If the technical solution contains a basic and a peak energy installation or engine, the binary matrices  $\mathbf{A}_p^b$  and  $\mathbf{F}_p^b$  are determined by summing up the binary matrices describing the consumption and by-production in the basic and peak ones, according to the principles of Boolean algebra [20]:

$$\mathbf{A}_p^b = \mathbf{A}_{pP}^b + \mathbf{A}_{pG}^b \quad (9.1)$$

$$\mathbf{F}_p^b = \mathbf{F}_{pP}^b + \mathbf{F}_{pG}^b \quad (9.2)$$

Based on the matrix  $\mathbf{A}_p^b$  the binary matrix  $\mathbf{Z}^b$  is formed describing the structure of the consumption of energy carriers in the set of technical solutions:

$$\mathbf{Z}^b = \left[ \mathbf{A}_1^b \mathbf{I}_0, \dots, \mathbf{A}_p^b \mathbf{I}_0, \dots, \mathbf{A}_m^b \mathbf{I}_0 \right] \quad (9.3)$$

where  $\mathbf{I}_0$  denotes the column vector with  $n_g$  unit elements; the multiplication of the matrix  $\mathbf{A}_p^b$  by the vector  $\mathbf{I}_0$  is in accordance with Boolean algebra.

As mentioned above, to each design topic an energy carrier is assigned, produced as the main product. Choosing one technical solution from each design topic, a set of all realizable variants of the energy management structure is elaborated and then subjected to a further analysis in order to choose the optimal variant. The number of all possible variants of energy management amounts to:

$$N = \prod_{t=1}^s m_t \quad (9.4)$$

where

- $t$     running number of the design topic,  
 $m_t$     number of technical solutions in the  $t$ th design topic,  
 $s$     number of design topics.



In order to find out all the possible variants of the energy management structure the binary column vector  $\mathbf{W}_l^b$  is set up, which is defined as follows:

$w_{pl} = 1$  if the  $p$ th solution belongs to the  $l$ th variant of the structure of energy management,

$w_{pl} = 0$  the opposite of the former,

where  $l = 1, 2, \dots, N$ , denotes the running number of the variant of the structure of energy management of the complex buildings.

The principle of the formation of the vector  $\mathbf{W}_l^b$  concerning each variant of energy management consists in assigning one technical solution to each design topic. The number of realizable variants of the energy management of complex buildings, determined in this way, is usually very high and a large number of the determined variants are economically a priori unjustified [6]. As an example may serve a variant in which electricity and heat are produced in several CHP units based on various technologies (e.g., a CHP unit with a piston combustion engine cooperating with a CHP unit based on a fuel cell). In this case there are two different cogeneration technologies. This does not exclude the so-called hybrid systems described in Chap. 4. In spite of the fact that the energy management of modern complex buildings is becoming more and more complex, the investor usually chooses only one kind of cogeneration technique. Although in modern complex buildings several parallelly operating CHP units are applied, they still work basing on the same technique. These remarks do not concern the production of energy carriers in basic and peak equipment. In order to exclude such cases (e.g., a combination of two different techniques of cogeneration) in further considerations they may be eliminated from the set of all variants of energy management applying the so-called criterion of technological compatibility. For this purpose the binary column vector  $\mathbf{L}_h^b$  is formed determining that group of energy installations which produce the same energy carriers by means of different technologies. The elements of this vector are defined as follows:

$\mathbf{L}_{uh} = 1$  if the  $u$ th energy installation belongs to the  $h$ th group of devices producing the same energy carriers applying different technologies,

$\mathbf{L}_{uh} = 0$  the opposite of the former case.

In order to identify the energy carriers produced together with other energy carriers in one common energy installation (e.g. electricity, hot water and cooling medium in BCHP unit) the binary column vector  $\mathbf{M}_s^b$  is formulated. The elements of this vector are determined as follows:

$\mathbf{M}_{is} = 1$  if the  $i$ th energy carrier belongs to the  $s$ th group of energy carriers produced together with other energy carriers in one common energy installation,

$\mathbf{M}_{is} = 0$  the opposite of the former case.

Based on the vectors  $\mathbf{L}_h^b$  and  $\mathbf{M}_s^b$  all those variants of energy management are eliminated which do not satisfy the criterion of technological compatibility, verifying in this way the set of variants.

In spite of the verification of the variants by means of the vectors  $\mathbf{L}_h^b$  and  $\mathbf{M}_s^b$  the respective vectors  $\mathbf{W}_l^b$  of the variants may include technical solutions not required to warrant cover the demands for energy carriers in the subsystem of consumers. For instance, replacing, an absorption refrigerator by a compression refrigerator, heat feeding the absorption aggregate is not required. In such cases, unnecessary technical solutions are eliminated from the vectors of the variants of the structure of the energy management of complex buildings. For this purpose the matrix  $\mathbf{Z}^b$  (Table 9.4 as an example cf. 9.7) is in compliance with the principles of Boolean algebra multiplied by the vector  $\mathbf{W}_l^b$  of the given variant, attaining in this way the vector  $\mathbf{Y}_l^b$  ( $\mathbf{Y}_l^b = \mathbf{Z}^b \mathbf{W}_l^b$ ) which determines the demand for the respective energy carriers resulting merely from the needs of the energy subsystem in the given variant of the energy management. In the next step, according to Boolean algebra, the vector  $\mathbf{O}^b$  is added to the vector  $\mathbf{Y}_l^b$ , expressing the demand for energy carriers in the subsystem of consumers. In this way the vector  $\mathbf{X}_l^b$  ( $\mathbf{X}_l^b = \mathbf{Y}_l^b + \mathbf{O}^b$ ) is obtained defining which energy carriers are consumed in the given variant. The vector  $\mathbf{X}_l^b$  is compared with the vector  $\mathbf{W}_l^b$  of the considered variant, attaining information whether in this variant those energy carriers are taken into account which are neither applied in the energy subsystem nor in the subsystem of consumers. If the vector  $\mathbf{X}_l^b$  includes a zero element for the given energy carrier and when simultaneously it results from the analysis of vector  $\mathbf{W}_l^b$  that it comprises a technical solution connected with the production or supply of this energy carrier, this a technical solution should be excluded from the given variant.

The binary matrices  $\mathbf{A}_P^b$ ,  $\mathbf{A}_G^b$ ,  $\mathbf{F}_P^b$ ,  $\mathbf{F}_G^b$  of interconnections describing the structure of the energy management of complex buildings are determined based on the following relations [20]:

$$\mathbf{A}_P^b = \Phi_P^b \mathbf{W}_l^b, \quad (9.5)$$

$$\mathbf{A}_G^b = \Phi_G^b \mathbf{W}_l^b, \quad (9.6)$$

$$\mathbf{F}_P^b = \Psi_P^b \mathbf{W}_l^b, \quad (9.7)$$

$$\mathbf{F}_G^b = \Psi_G^b \mathbf{W}_l^b, \quad (9.8)$$

where

$$\Phi_P^b = [\mathbf{A}_{1P}^b, \dots, \mathbf{A}_{pP}^b, \dots, \mathbf{A}_{mP}^b], \quad (9.9)$$

$$\Phi_G^b = [\mathbf{A}_{1G}^b, \dots, \mathbf{A}_{pG}^b, \dots, \mathbf{A}_{mG}^b], \quad (9.10)$$

$$\Psi_P^b = [\mathbf{F}_{1P}^b, \dots, \mathbf{F}_{pP}^b, \dots, \mathbf{F}_{mP}^b], \quad (9.11)$$

**Table 9.4** Binary matrix  $Z^b$ -consumption of energy carriers in set of designs

Energy carrier/design	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1 Electricity	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2 Heat	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
3 Cold	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0
4 Hot process water for abs. chiller I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
5 Cooling medium I 6/12 °C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6 Hot process water I 60/45°C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7 Hot process water II 85/55°C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8 Hot tap water	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9 Air from air conditioning unit	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10 Ventilation air	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11 Drinking water	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
12 Natural gas	0	0	1	1	0	1	1	0	1	0	1	1	1	1	1	0	0	1	0	1	1	0	0	0	0	0	0
13 Heating oil	0	0	0	0	1	0	1	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
14 Diesel oil	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

$$\Psi_G^b = [\mathbf{F}_{1G}^b, \dots, \mathbf{F}_{pG}^b, \dots, \mathbf{F}_{mG}^b]. \quad (9.12)$$

where the binary matrices of interbranch connections  $\mathbf{A}_{pP}^b$ ,  $\mathbf{A}_{pG}^b$ ,  $\mathbf{F}_{pP}^b$  as well as  $\mathbf{F}_{pG}^b$  being the data base, determine the structure of demands and the by-production of the energy carriers required for the respective design solutions concerning the basic and peak installations.

The input–output binary matrix  $\mathbf{A}^b + \mathbf{F}^b$  is determined based on the relation:

$$\mathbf{A}^b + \mathbf{F}^b = \mathbf{A}_P^b + \mathbf{A}_G^b + \mathbf{F}_P^b + \mathbf{F}_G^b \quad (9.13)$$

The summation is in agreement with the principles of Boolean algebra.

### 9.3 Structural Analysis of the Binary Input–Output Matrix

A structural analysis permits the degree of complexity of interconnections between the energy branches to be assessed and the input–output matrix to be transformed in order to obtain a structure of this matrix better adapted for further calculations. This means a transformation of the binary input–output matrix approximated to the upper triangular matrix as far as possible. This procedure is called minimization of the number of nonzero elements below the main diagonal of the input–output matrix, because these elements determine the feedback loops which increase the complexity of numerical calculations [10]. The structural analysis is performed by means of the binary input–output matrix  $(\mathbf{A}^b + \mathbf{F}^b)$ .

The following characteristic stages of structural analysis can be distinguished:

- decomposition of the energy subsystem,
- separation of strongly coherent subsystems,
- sequence of balancing the energy branches.

Decomposition consists in the division of the energy management system of complex buildings into subsystems of a lower order, comprising the production of one energy carrier or a group of energy carriers. First of all, in the binary matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  the “output”, “centre”, and “input” blocks are separated. Energy branches belonging to the output block do not influence other energy branches, but they themselves are affected by the energy subsystem. Energy carriers produced in branches belonging to this block are used only in the subsystem of consumers. In the matrix  $\mathbf{A}^b + \mathbf{F}^b$  the branches of the output block correspond to the zero rows. Energy branches belonging to the input block are not influenced by the energy subsystem, but they affect other energy branches. These branches are characterized by the fact that in them no other energy carriers are consumed. In the matrix  $\mathbf{A}^b + \mathbf{F}^b$  the zero columns correspond to the branches belonging to this block. The remaining branches belonging to the center block are characterized by interconnections resulting from the consumption and by-production of energy carriers.

Feedback connections exist between the branches belonging to this group. The separation of these three mentioned groups of branches results in the division of the matrix  $\mathbf{A}^b + \mathbf{F}^b$  into blocks. The division of energy branches into output, center, and input blocks provides the first approximation of the input–output matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  to its upper triangular form.

Further in the course of this structural analysis strongly coherent subsystems can be distinguished among the branches belonging to the center block [1]. The set of branches is a strongly coherent subsystem if any given branch of this set is connected with every other branch of this set. Thus, the center submatrix is transformed into a block matrix. Interconnections between branches belonging to various strongly coherent subsystems are series connections. Feedback connections exist only between branches belonging to the same strongly coherent subsystem. The transformation of the input–output matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  to the form of the upper triangular block containing the least number of elements situated below the main diagonal determines the sequence in the choice of the energy equipment.

In the case of already existing energy management systems, the matrix equation of the energy balance is usually solved by means of the inverse matrix  $(\mathbf{I} - \mathbf{A} + \mathbf{F})^{-1}$ . In the course of preliminary design, the values of the elements of the matrices  $\mathbf{A}_G$ ,  $\mathbf{A}_P$ ,  $\mathbf{F}_G$ ,  $\mathbf{F}_P$ , are unknown. Thus, the set of balance equations of energy carriers is solved separately for each branch in the order determined by the structural analysis ensuring the best effectivity of calculations. And this is why the matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  should be transformed into a matrix most similar to the upper triangular matrix. For this purpose this sequence of branches in the submatrix concerning the strongly coherent system is to be chosen, so that the number of elements below the main diagonal of the matrix will be at a minimum.

In order to separate strongly coherent subsystems in the center group, the following matrix has been determined:

$$\mathbf{C} = \sum_{s=1}^r (\mathbf{A}^b + \mathbf{F}^b)_c^s \quad (9.14)$$

where summing up of the successive powers of the matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  is continued until the condition is met, where for any  $r \leq n_c$ :

$$\sum_{s=1}^r (\mathbf{A}^b + \mathbf{F}^b)_c^s = \sum_{s=1}^{r+1} (\mathbf{A}^b + \mathbf{F}^b)_c^s = \dots = \sum_{s=1}^{n_c} (\mathbf{A}^b + \mathbf{F}^b)_c^s \quad (9.15)$$

where

$n_c$  the number of energy carriers belonging to the center group,  
 $(\mathbf{A}^b + \mathbf{F}^b)_c$  binary matrix of interconnections belonging to the center block. In Eqs. (9.14) and (9.15) the principles of Boolean algebra are obligatory.

Matrix  $\mathbf{C}^b$  is defined as follows:

$$\begin{aligned} c_{ij}^b &= 1 && \text{if between the branches } i \text{ and } j \text{ of the center block exist direct or indirect} \\ &&& \text{connections,} \\ c_{ij}^b &= 0 && \text{the opposite of the former one.} \end{aligned}$$

The input–output matrix  $(\mathbf{A}^b + \mathbf{F}^b)_c$  informs us whether direct connections between the branches (energy carriers) “i” and “j” do exist, whereas the matrix  $\mathbf{C}^b$  also provides information about indirect connections between these branches realized by means of branches belonging to the center block.

Next the matrix of intersection  $\mathbf{W}$  is deduced from the equation:

$$\mathbf{W} = \mathbf{C} \cap \mathbf{C}^T \quad (9.16)$$

which may be defined as follows:  $w_{ij}^b = 1$  if  $c_{ij}^b = c_{ji}^b = 1$ ,  $w_{ij}^b = 0$  the opposite of the former one.

The matrix  $\mathbf{W}$  comprises nonzero elements only in the case when nonzero elements occur in the matrix  $\mathbf{C}$  and matrix  $\mathbf{C}^T$ . In the matrix  $\mathbf{W}$  one-sided connections are reduced to zero. The matrix  $\mathbf{W}$  is a matrix with diagonally arranged blocks. The respective blocks along the main diagonal represent strongly coherent subsystems.

The transformation of the binary input–output matrix into the form of the upper triangular block matrix with a minimum number of elements below the main diagonal consists in an adequate arrangement of the rows and columns in the input–output matrix, applying for this purpose the algorithms of the topological classification. This matrix determines the sequence of balancing the energy demands and the choice of energy engines and installations.

## 9.4 Mathematical Optimization Model of the Energy Balance

The aim of preliminary design is to choose the optimal variants of the energy management system. The criterion of the energy management is the maximum of the annual economic effect of operation of complex buildings for which the structure of energy management has been chosen. In the case of the assumed demand for energy carriers in the subsystem of consumers of complex buildings this is reduced to a minimization of the annual costs of operation of the energy subsystem. The objective function takes the following form according to Eqs. (3.19)–(3.21):

$$\begin{aligned} C_a &= (\rho_p + \beta_p)\mathbf{I}_p + (\rho_G + \beta_G)\mathbf{I}_G + (\rho_{DG} + \beta_{DG})\mathbf{I}_{DG} + \alpha_P\dot{\mathbf{P}}_n + \alpha_G\dot{\mathbf{G}}_n \\ &\quad + \alpha_{DG}\dot{\mathbf{D}}_{Gn} + \varepsilon_P\mathbf{P} + \varepsilon_G\mathbf{G} + \varepsilon_{DG}\mathbf{D}_G + \mathbf{k}_{DD}\mathbf{D}_D \Rightarrow \min \end{aligned} \quad (9.17)$$

where

- $C_a$  total annual cost,  
 $\rho$  -row vector of the annual capital recovery factor,  
 $\beta$  -row vector of the rates of fixed costs dependent on the capital expenditure,  
 $I$  -column vector of capital expenditure,  
 $\alpha$  -row vector of the indices of labor costs,  
 $\varepsilon$  -row vector of the indices of exploitation costs including environment taxes; in the case of supplementary supplies also cost of purchase,  
 $\dot{P}_n$  -column vector of the power rating of basic equipment,  
 $\dot{G}_n$  column vector of the power rating of peak equipment,  
 $\dot{D}_{Gn}$  column vector of nominal supplementary supplies from outside,  
 $P$  column vector of the annual production by basic equipment,  
 $G$  column vector of the annual production by peak equipment,  
 $D_G$  column vector of the annual supplementary supply of energy carriers,  
 $k_{DD}$  -row vector of unit costs of energy carriers entirely supplied from outside,  
 $D_D$  -column vector of annual supply of energy carriers entirely from outside.

The indices “ $P$ ”, “ $G$ ”, “ $DG$ ” and “ $DD$ ” concern, respectively, the basic and peak parts of production, supplementary supplies and energy carriers entirely supplied from outside.

The relations between the annual production  $G_i$ ,  $P_i$ ,  $D_i$  and power ratings  $\dot{G}_{ni}$ ,  $\dot{P}_{ni}$ ,  $\dot{D}_{Gni}$  are:

$$\dot{G}_{ni} = \frac{1}{\tau_n} \int_0^{\tau_0} \dot{G}_i(\tau) d\tau \quad (9.18)$$

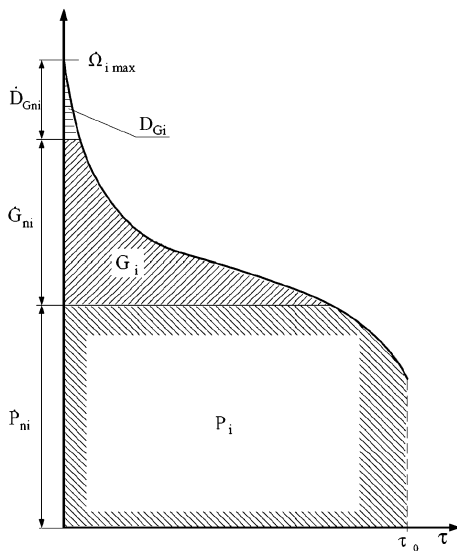
$$\dot{P}_{ni} = \frac{1}{\tau_n} \int_0^{\tau_0} \dot{P}_i(\tau) d\tau \quad (9.19)$$

$$\dot{D}_{Gni} = \frac{1}{\tau_n} \int_0^{\tau_0} \dot{D}_{Gi}(\tau) d\tau \quad (9.20)$$

where  $G_i(\tau)$ ,  $P_i(\tau)$  and  $D_{Gi}(\tau)$  denote functions describing the duration curves and  $\tau_n$ ,  $\tau_0$  denote the annual duration of work with the power rating load and the annual duration of operation.

Decision variables are vectors of the power ratings  $\dot{G}_n$ ,  $\dot{P}_n$ ,  $\dot{D}_{Gn}$ . The vector of peak production  $G$  and basic production  $P$ , as well as the vector of external supplementary supplies  $D_G$  result from the choice of the optimal power rating and nominal capacities and also from the duration curves of the total demand for energy carriers (Fig. 9.1).

**Fig. 9.1** Duration curve concerning annual demand for the “*i*th” energy carrier



The global equality constraint is a matrix equation of the energy carriers balance quoted in Chap. 6 concerning the descriptive mathematical model. They are equality constraints in the optimization task. Local constraints concerning the respective energy branches result from the maximum demand for each energy carrier and from the limitation of external supplies and capital expenditures, if required:

$$\dot{P}_{ni} + \dot{G}_{ni} + \dot{D}_{ni} \leq \dot{\Omega}_{i \max} \quad (9.21)$$

$$\dot{D}_{Gni} \leq \vartheta_{DGi} \text{ and } \dot{D}_{Dni} \leq \vartheta_{DDi} \quad (9.22)$$

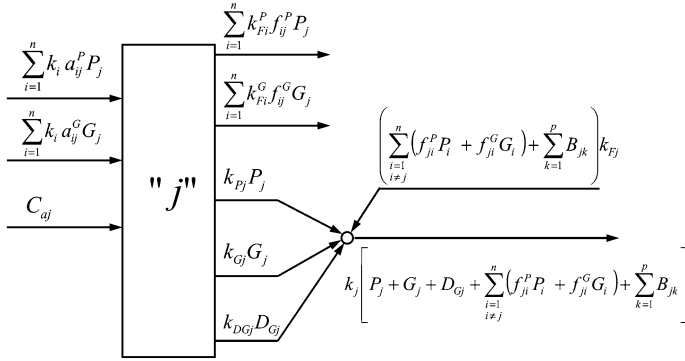
$$I_{Pi} + I_{Gi} + I_{Di} \leq I_i \quad (9.23)$$

where

- $\dot{\Omega}_{i \max}$  maximum demand for the *i*th energy carrier,
- $\vartheta_{DGi}$  limit of external supplementary supplies of the *i*th energy carrier,
- $\dot{D}_{Dni}$  nominal flux of the *i*th energy carrier entirely supplied from outside,
- $\vartheta_{DDi}$  limit of the *i*th energy carriers entirely supplied from outside,
- $I_i$  limit of capital expenditures concerning the *i*th energy carriers.

In the case of external supplementary supplies capital expenditures and costs of exploitation have been taken into account connected with the change of thermodynamic parameters and distribution of external supplies. As far as energy carriers supplied entirely from outside are concerned it has been assumed that they are charged only with the costs of their purchase.





**Fig. 9.2** Auxiliary diagram used for the formulation of the objective function of the energy branch

## 9.5 Decomposition of the Global Optimization Task

In order to solve the global optimization task (9.17) we must know the duration curves of the demand for each energy carrier, as well as the technical coefficients of consumption and by-production of energy carriers. These are determined based on the energy characteristics depending on both the power rating and the load of energy engine and equipment. Changes in the load are expressed by the duration curves, but in order to construct the duration curves the technical coefficients must be known. Thus, the construction of duration curves of the demand for energy carriers, the determination of the technical coefficients of consumption and by-production of energy carriers, the choice of the power ratings of machines and energy equipment, and also the nominal amount of supplies are mutually connected tasks. Their solution requires the decomposition of the global optimization task. For this purpose Lagrange's method of decomposition is applied [10]. Taking into account the constraint Eq. (6.12) (Chap. 6) and disregarding the constant terms **B** and **C** which do not influence the result of optimization, the objective function (9.17) takes the following form

$$\mathbf{L} = C_a + \lambda(\mathbf{A}_G \mathbf{G} + \mathbf{A}_P \mathbf{P} - \mathbf{G} - \mathbf{P} - \mathbf{F}_G \mathbf{G} - \mathbf{F}_P \mathbf{P} - \mathbf{D}_G) \quad (9.24)$$

where  $\lambda$  denotes the raw vector of Lagrange multipliers.

The determination of the vector  $\lambda$  requires knowledge of the coordinating procedure warranting the compatibility of the local optimum of the respective branches with the global one. In order to determine this procedure in the analysis under consideration, the objective function is formulated concerning any  $j$ th energy branch. An auxiliary balance diagram for the formulation of the objective function and further considerations is presented in Fig. 9.2. The main and by-production are the useful results of the operation of this branch. Various unit costs of the production of both kinds of main production (basic and peak production) have been taken into account. Besides the quantities in the global objective function the

expenditure in this case also comprises the costs of consumed energy carriers produced in other branches of the energy subsystem. In this case, the average costs of the production of energy carriers in the energy subsystem are applied [21]. The external supplies (supplementing supplies—e.g., electricity, and only those from outside—e.g., fuel) are taken into account in  $C_{aj}$ . In this case, the expenditure comprises fixed and variable costs connected with the preparation of the energy carrier supplied from outside to be used in the complex buildings (e.g., costs connected with the operation of the thermal center). Figure 9.2 also presents the summing point for the determination of the average cost of the energy carrier [21].

The objective function concerning the considered  $j$ th branch of the energy subsystem is expressed as follows:

$$\varphi_j = C_{aj} + \sum_{i=1}^n k_i (a_{ij}^P P_j + a_{ij}^G G_j) - \sum_{i=1}^n k_{Fi} f_{ij}^P P_j - \sum_{i=1}^n k_{Fi} f_{ij}^G G_j \Rightarrow \min \quad (9.25)$$

where:

$$\begin{aligned} C_{aj} = & (\rho_{Pj} + \beta_{Pj}) I_{Pj} + (\rho_{Gj} + \beta_{Gj}) I_{Gj} + (\rho_{DGj} + \beta_{DGj}) I_{DGj} + \alpha_{Pj} \dot{P}_{nj} \\ & + \alpha_{Gj} \dot{G}_{nj} + \alpha_{DGj} \dot{D}_{Gnj} + \varepsilon_{pj} P_j \\ & + \varepsilon_{Gj} G_j + \varepsilon_{DGj} D_{Gj} + \sum_{l=n+1}^m k_{DDl} a_{lj}^{PD} P_j + \sum_{l=n+1}^m k_{DDl} a_{lj}^{GD} G_j \end{aligned} \quad (9.26)$$

where (both in Fig. 9.2 and Eqs.(9.25, 9.26):

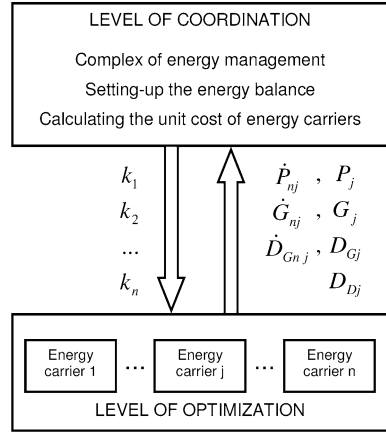
$C_{aj}$	part of the objective function (9.17) concerning the $j$ th energy branch,
$k_i$	average unit cost of the $i$ th energy carrier,
$k_{Pj}$	unit cost of the basic part of production of the $j$ th energy carrier,
$k_{Fj}, k_{Fi}$	unit cost of the by-production of the $j$ th or $i$ th energy carrier,
$k_{Gj}$	unit cost of the peak part of production of the $j$ th energy carrier,
$k_{DDl}$	unit cost of energy carriers entirely supplied from outside.

The descriptions of remaining values in Eq. (9.26) correspond to descriptions of the respective vectors in Eq. (9.17). The indices “i” and “j” are applied in the denominations interchangeably.

The applied method of decomposition reduces the optimization task to an iterative procedure [10]. In the successive iterative step on the upper level of the hierarchical scheme Lagrange’s multipliers are determined by an adequate coordinating procedure. Then based on the assumed values of Lagrange’s multipliers, every subsystem on the lower level of the hierarchical scheme is autonomously optimized [10]. In the case under consideration the level of the subsystem (energy branches) the power ratings of the basic and peak equipment are determined as well as the nominal quantities of supplies of energy carriers from outside (Fig. 9.3).

In Eq. (9.25) concerning the objective function on the level of optimization of the respective energy branch the cost  $k_i$  and  $k_{Fi}$  of energy carriers are fixed. The following terms are also fixed:

**Fig. 9.3** Diagram of the Lagrange's method of the decomposition



$$k_j(P_j + G_j + D_{Gj}) = \text{const} \quad (9.27)$$

$$(k_j - k_{Fj}) \sum_{i=1, i \neq j}^n (f_{ji}^P P_i + f_{ji}^G G_i) = \text{const} \quad (9.28)$$

The term (9.27) is constant, because in successive iteration the global demand for the energy carrier “j” is fixed. The division of the global demand for energy carriers between the energy produced in the energy management of complex buildings itself and the external supply is solved on the level of optimization. The term (9.28) is also constant in the given iteration due to the summation with regard to “i”, except  $i = j$ . Hence, the terms (9.27) and (9.28) may be included in the objective function (9.25) without influencing the results of optimization. After the summation of the objective function (9.25) for all the energy branches considered on the level of optimization with the additionally included terms (9.27) and (9.28) we obtain:

$$\begin{aligned} \sum_{j=1}^n \varphi_j &= \sum_{j=1}^n C_{aj} + \sum_{j=1}^n \sum_{i=1}^n k_i (a_{ij}^P P_j + a_{ij}^G G_j) \\ &\quad - \sum_{j=1}^n \sum_{i=1}^n k_{Fj} f_{ij}^P P_j - \sum_{j=1}^n \sum_{i=1}^n k_{Fj} f_{ij}^G G_j \\ &\quad - \sum_{j=1}^n k_j (P_j + G_j + D_{Gj}) \\ &\quad - \sum_{j=1}^n k_j \sum_{i=1}^n (f_{ji}^P P_i + f_{ji}^G G_i) \\ &\quad + \sum_{j=1}^n k_{Fj} \sum_{i=1}^n f_{ji}^P P_i + \sum_{j=1}^n k_{Fj} \sum_{i=1}^n f_{ji}^G G_i \end{aligned} \quad (9.29)$$

Reducing similar terms in Eq. (9.29) and using the matrix notation we obtain:

$$\sum_{j=1}^n \varphi_j = C_a + \mathbf{k}^T (\mathbf{A}_G \mathbf{G} + \mathbf{A}_P \mathbf{P} - \mathbf{G} - \mathbf{P} - \mathbf{F}_G \mathbf{G} - \mathbf{F}_P \mathbf{P} - \mathbf{D}_G) \quad (9.30)$$

where  $\mathbf{k}$  denotes the column vector of the average unit costs of energy carriers.

Comparing Eqs. (9.30), and (9.24), we can write:

$$L = \sum_{j=1}^n \varphi_j \quad (9.31)$$

and

$$\lambda = \mathbf{k}^T \quad (9.32)$$

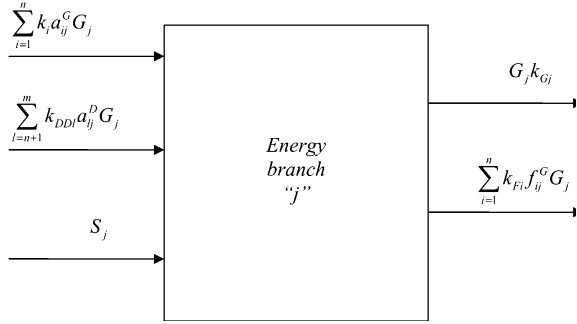
Thus, the Lagrange multipliers are equal to the unit costs of energy carriers produced in energy subsystem of complex buildings. Therefore, the matrix method of calculating the unit costs of energy carriers is a coordinating procedure in the decomposition model for the preliminary design of the energy management of complex buildings [21].

In the first approximation of the iterative algorithm of decomposition the vectors  $\mathbf{k}$  and  $\mathbf{k}_F$  of unit costs of the energy carriers is assumed as well as the technical coefficients of feedback character. Having solved the task of optimization concerning all the branches Eq. (9.25), and taking into account local restrictions we return to the level of coordination where the corrected balance of energy carriers is set up by means of linear mathematical model [15, 16], and the corrected vector of unit costs of energy carriers is determined in compliance with the coordinating procedure. The corrected vector of unit costs is then applied to optimize the branches in the successive iteration. The end of the iterative procedure results from the assumed accuracy in the calculation of the unit cost of energy carriers.

## 9.6 Matrix Method for Calculating the Unit Costs of Energy Carriers

The complexity of energy management due to feedback connections allows the unit costs of the production of energy carriers to be determined merely by means of the method of successive approximation. Thus, for instance, the unit cost of electricity requires knowledge of the unit cost of the driving agent (steam driving the steam turbine or combustion gases driving the gas turbine) which again depends on the a priori unknown unit cost of electricity. An additional difficulty arises from problems connected with the division of costs in cogeneration processes. Therefore, the application of the matrix method for determining unit costs

**Fig. 9.4** Balance of costs for the energy branch “j” in the case of peak part production



in energy management, based on the same principles as the mathematical model of the energy balance, provides a solution of this problem [18]. An indispensable condition of its application is a priori knowledge of the energy balance prepared, based on the mathematical model of the energy management of complex buildings.

The energy carrier consumed in complex buildings may be the main product of basic and peak equipment, a by-product or supply from outside. For this reason, unit costs of the basic and peak part of the main production, by-production of energy carriers, and externally supplied energy carriers are to be distinguished. As far as the consumption of energy carriers is concerned, the average unit cost is to be used, calculated as the weighted average taking into account the cost of transport, changes of the parameters, and the cost of distribution of the energy carriers.

The balance equation of costs concerning the peak part production of the  $j$ th energy branch (Fig. 9.4.) takes the form:

$$\sum_{i=1}^n k_i a_{ij}^G G_j + \sum_{l=n+1}^m k_{DDl} a_{lj}^{GD} G_j + S_j = G_j k_{Gj} + \sum_{i=1}^n k_{Fi} f_{ij}^G G_j \quad (9.33)$$

where

- $k_i$  weighted average unit cost of the “ $i$ th” energy carrier,
- $k_{DDl}$  unit cost of the “ $l$ th” energy carrier entirely supplied from outside,
- $k_{Gj}$  unit cost of the “ $j$ th” energy carrier produced by peak equipment,
- $k_{Fi}$  unit cost of the “ $i$ th” energy carrier produced as a by-product,
- $S_j$  arbitrary fixed cost.

This equation concerns the case when the  $j$ th energy carrier is produced in the peak equipment accompanied by the by-production of other energy carriers. The term  $S_j$  in the balance Eq. (9.33) comprises all the remaining components of costs beside the cost of the energy carriers. As  $S_j$  includes mainly the components of fixed costs, it has been called arbitrary fixed cost.

The average unit cost  $k_i$  of the  $i$ th energy carrier is determined by means of the equation:

$$k_i = r_{Pi}k_{Pi} + r_{Gi}k_{Gi} + r_{Fi}k_{Fi} + r_{DGi}k_{DGi} \quad (9.34)$$

where  $r_{Pi}, r_{Gi}, r_{Fi}, r_{DGi}$  denote the share of the basic and peak part of main production, by-production, and external supplementary supply in the global input of the “ $i$ th” energy carrier and  $k_{Pi}, k_{Gi}, k_{Fi}, k_{DGi}$  denote the unit cost of basic and peak production, by-production, and supplementary supplies.

The set of balance equations of the costs of the peak part of the production concerning all the considered energy carriers takes in matrix notation the following form:

$$(\mathbf{A}_G \mathbf{G}^d)^T \mathbf{k} + (\mathbf{A}_{GD} \mathbf{G}^d)^T \mathbf{k}_{DD} + \mathbf{S}_G = \mathbf{G}^d \mathbf{k}_G + (\mathbf{F}_G \mathbf{G}^d)^T \mathbf{k}_F \quad (9.35)$$

where

- $\mathbf{k}$  vector of weighted average unit costs of energy carriers,
- $\mathbf{k}_{DD}$  vector of the unit costs of energy carriers entirely supplied from outside,
- $\mathbf{S}_G$  vector of arbitrary fixed costs concerning the peak part of the production,
- $\mathbf{k}_G$  vector of the unit costs of peak part of the production,
- $\mathbf{k}_F$  vector of the unit costs of by-production,

“ $d$ ” denotes the construction of a diagonal matrix from the column vector.

Similarly, the balancing of the production costs of the basic part of the production takes the following form:

$$(\mathbf{A}_P \mathbf{P}^d)^T \mathbf{k} + (\mathbf{A}_{PD} \mathbf{P}^d)^T \mathbf{k}_{DD} + \mathbf{S}_P = \mathbf{P}^d \mathbf{k}_P + (\mathbf{F}_P \mathbf{P}^d)^T \mathbf{k}_F \quad (9.36)$$

where

- $\mathbf{k}_P$  vector of the unit costs of basic part of the production,
- $\mathbf{S}_P$  vector of arbitrary fixed costs concerning the basic part of the production.

The vector of weighted average unit costs of energy carriers is expressed by the equation:

$$\mathbf{k} = \mathbf{r}_P^d \mathbf{k}_P + \mathbf{r}_G^d \mathbf{k}_G + \mathbf{r}_F^d \mathbf{k}_F + \mathbf{r}_{DG}^d \mathbf{k}_{DG} \quad (9.37)$$

where

- $\mathbf{r}_P^d$  diagonal matrix of the shares of basic part of the main production,
- $\mathbf{r}_G^d$  diagonal matrix of the shares of peak part of the main production,
- $\mathbf{r}_F^d$  diagonal matrix of the shares of by-production,
- $\mathbf{r}_{DG}^d$  diagonal matrix of the shares of supplementary supplies of energy carriers,
- $\mathbf{k}_{DG}$  vector of the unit costs of supplementary supplies.

Equations (9.35), (9.36), and (9.37) constitute the algorithm of the matrix method for calculating the unit costs of energy carriers. The unit costs of by-production are determined using the method of avoided costs [12]. Unit costs of supplementary supplies, as well as unit costs of energy carriers entirely supplied from outside, are input data known a priori.

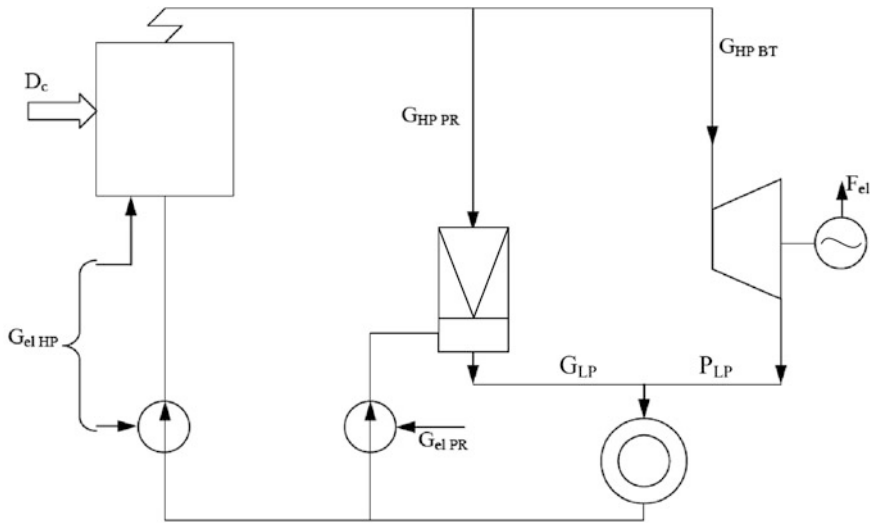


Fig. 9.5 CHP plant with back-pressure turbine

The presented matrix method permits the unit costs of energy carriers concerning the complex energy management system to be determined with a systems approach. It is pertinent independently of the applied methods of dividing the costs in cogeneration and coupled processes. It is of advantage when the applied auxiliary equations, resulting mainly from the principles of divided costs, do not derange the linearity of the set of equations, only due to the solution of the set of balance equations of the costs.

#### Example of unit costs calculations

An example is a CHP unit with a back-pressure turbine (Fig. 9.5). The back-pressure steam is the basic part of the production of process steam. Electricity is a by-product of the CHP unit produced on the flux of back-pressure steam (process steam) and it is also provided by the national electro-energy system.

The balance equations of the costs concerning high pressure steam, peak, and basic production of low-pressure steam for heating purposes take the forms:

$$D_c k_c + G_{elHP} k_{el} + S_{HP} = (G_{HP PR} + G_{HP BT}) k_{HP} \quad (a)$$

$$G_{HP PR} k_{HP} + G_{elPR} k_{el} + S_{PR} = G_{LP} k_{GLP} \quad (b)$$

$$G_{HP BT} k_{HP} + S_{BT} = P_{LP} k_{PLP} + F_{el} k_{Fel} \quad (c)$$

where

$G_{HP BT}$  consumption of high-pressure steam in back-pressure turbine,  
 $G_{HP PR}$  consumption of high-pressure steam by pressure-reducing valve station,  
 $D_c$  supply of hard coal,

$k_c$	unit cost of hard coal,
$G_{el\ PR}$	consumption of electricity by steam boiler,
$G_{el\ PR}$	consumption of electricity by pressure-reducing valve station,
$G_{LP}$	peak part production of low-pressure steam,
$P_{LP}$	basic part production of low-pressure steam (back-pressure steam),
$k_{G\ LP}$	unit cost of peak part of the production of low-pressure steam,
$k_{P\ LP}$	unit cost of basic part of the production of low-pressure steam,
$k_{HP}$	unit cost of high-pressure steam,
$k_{el}$	weighted average unit cost of electricity,
$k_{FeI}$	unit cost of electricity produced as a by-product,
$S_{HP}, S_{PR}, S_{BT}$	arbitrary fixed cost of the steam boiler, pressure-reducing valve, and back pressure turbine, respectively, excluding the costs of energy carriers,
$F_{el}$	by-production of electricity.

The weighted average unit cost of electricity results from:

$$k_{el} = \frac{F_{el}}{F_{el} + D_{el}} k_{FeI} + \frac{D_{el}}{F_{el} + D_{el}} k_{DeI} \quad (d)$$

where  $D_{el}$  external supply of electricity,  $k_{DeI}$  unit cost of electricity supplied from outside.

Based on the principle of avoided costs concerning the cogeneration system we can write:

$$k_{FeI} = k_{DeI} \quad (e)$$

Hence:

$$k_{el} = k_{DeI} \quad (f)$$

Based on Eq. (a) we can calculate:

$$k_{HP} = \frac{D_c k_c + G_{el\ HP} k_{DeI} + S_{HP}}{G_{HP\ PR} + G_{HP\ BT}} \quad (g)$$

and from Eq. (b) and (c) we get:

$$k_{G\ LP} = \frac{G_{HP\ PR}}{G_{LP}} \left( \frac{D_c k_c + G_{el\ HP} k_{DeI} + S_{HP}}{G_{HP\ PR} + G_{HP\ BT}} \right) + \frac{G_{el\ PR}}{G_{LP}} k_{DeI} + \frac{S_{PR}}{G_{LP}} \quad (h)$$

$$k_{P\ LP} = \frac{G_{HP\ BT}}{P_{LP}} \left( \frac{D_c k_c + G_{el\ HP} k_{DeI} + S_{HP}}{G_{HP\ PR} + G_{HP\ BT}} \right) + \frac{S_{BT}}{P_{LP}} - \frac{F_{el}}{P_{LP}} k_{DeI} \quad (i)$$

The unit costs of external supplies, viz.  $k_{DeI}$  concerning the supplementary supply of electricity and  $k_c$  concerning the supply of coal are assumed to be imposed by the suppliers. Arbitrary fixed costs  $S_{HP}$ ,  $S_{PR}$ , and  $S_{BT}$  are known. The



remaining values in Eqs. (h) and (i) result from the energy and mass balance equations concerning CHP plant (Fig. 9.5).

## 9.7 Example of Calculations Concerning the Application of a Descriptive Model of Complex Buildings

### 9.7.1 Scenario of the Energy Management of the Analyzed Complex Buildings

As an example of complex buildings a typical office building has been chosen. The seven-floor building provides main office services. The floor area of the offices in the building amounts to 8,000 m<sup>2</sup>. The building also includes car park floors, which are neither heated nor cooled. In this building, there are two types of rooms, viz., offices with fan-coil units and auxiliary rooms with a traditional central heating system.

The fan-coil system is one of the air-conditioning systems used in the building. Fan-coil units are placed in each office which needs to be heated or cooled. This system uses in-room units containing such components as a fan, heating and cooling coils, filters and controls. This is a four-pipe system utilizing two independent coil, one for heating and one for cooling. Cooling and heating valves for controlling the coil capacities are installed with their controls in the rooms. A central plant delivers hot or cold water to the fan units. The mechanical ventilation system operates only during office hours. The main air-conditioning unit for the building delivers a suitable quantity of air to all rooms. Individual temperature conditions are ensured by fan-coil units in the offices and by the central heating system in auxiliary rooms. The installation of hot process water supplies heat to the preheater in the air-conditioning unit and to the offices via the fan-coil units.

Car parking is situated on four floors. These floors are only ventilated by a mechanical extract ventilation system. The car park consumes electricity only for lighting, controls, and monitoring.

The following energy carriers need to be supplied to the subsystem of consumers:

- cooling medium (water-glycol) 6/12 °C,
- hot process water (for the central heating system and for the air-conditioning unit) 85/55 °C,
- hot tap water,
- hot process water 60/45 °C (for fan-coil units),
- air from the air-conditioning unit.

The subsystem of consumers includes:

- office rooms with fan-coil air-conditioning,
- auxiliary rooms with a traditional central heating system,

- standard building equipment, such as an emergency systems, fire sensors, and external lighting,
- car park with a mechanical extract ventilation system.

For the energy management of exemplary complex buildings it was suggested that a small cogeneration unit is to be applied, equipped with a piston combustion engine, a gas microturbine, or a fuel cell. It was decided to consider variants of the energy management which satisfy the condition that one cogeneration technique is assigned to every respective variant. For this purpose the criterion of technical compatibility has been applied. Also, the application of the “trigeneration” technique, using the heat from the cogeneration unit in an absorption chiller fed with hot water, was taken into consideration. The possibility of supplying electricity and heat from outside and the production of the cooling agent in a compression refrigerator, were also considered.

The subsystem of consumers in the analyzed complex buildings requires process hot water on two temperature levels, as well as hot tap water. Process water feeds the heating installations and the preheater in the air-conditioning center. Three cooling agents with a different temperature have been taken into account, respectively for the air-conditioning unit, the cooler and store rooms. The production of the cooling agent in an absorption chiller fed with natural gas is presumed to be possible. Moreover, the installation of the following peak equipment has been taken into consideration: a water-heater boiler fed with natural gas or fuel oil, and a compressor refrigerator driven by electricity.

Table 9.1 contains a general list of energy carriers and, among others, the vector  $\mathbf{O}^b$  describing the structure of the demand for energy carriers by the subsystem of consumers. Besides energy carriers consumed by the subsystem of consumers (expressed by the binary vector  $\mathbf{O}^b$ ) the general specification contains energy carriers, the production of which results from interbranch connections in the energy subsystem. The general specification contains ten energy carriers produced as major products and four energy carriers supplied from outside. Nonzero elements of the vector  $\mathbf{O}^b$  provide information about the direct consumption of energy carriers in the subsystem of consumers. Zero elements of the vector  $\mathbf{O}^b$  concern energy carriers which are potentially consumed only in the energy subsystem. Nonzero elements of the vectors  $\mathbf{B}_G^b$  and  $\mathbf{B}_D^b$  represent energy carriers produced in the energy subsystem of complex buildings and those which are supplied from outside.

The preliminary design of the energy subsystem of the considered complex building concerns the modernization of the energy management. Instead of supplying heat and electricity, the CHP unit has been suggested. Based on the general specification of energy carriers (Table 9.1) a list of energy equipment has been drawn up (Table 9.2). This list comprises technically available and economically feasible energy equipment selected as needed by the subsystem of consumers.

### 9.7.2 Elaboration of the Set of Admissible Variants

On the basis of the scenario of the energy management of the analyzed complex buildings and information provided in Tables 9.1 and 9.2, sets of projects and designs have been formulated (Table 9.3).

Further on, they form the basis for setting up all the realizable variants of the energy management of complex buildings. This set is then verified according to the criterion of compatibility of the technologies and the elimination of unnecessary technical solutions.

The set of the project and designs is presented as a binary matrix  $\Pi$  collated in Table 9.3. This matrix comprises all the projects and designs to be realized in the exemplary building. Ten projects and 27 designs have been distinguished. The rows in the matrix  $\Pi$  correspond to the respective designs, and the columns to the respective energy equipment in the analyzed complex buildings. The last column concerns the supply of energy carriers from outside. Thus, for example, project No 1, comprising four designs, concerns the production or supply of electricity. Design No 1 (first row in Table 9.3) consists in supplying electricity from outside (only supply without own production). The next three designs comprise the successive applications of a CHP unit with a piston engine fed with diesel oil, a CHP unit with a gas microturbine fed with natural gas and a CHP unit with a fuel cell together with a supplementary supply of electricity from the electrical grid.

Each one of the 27 designs is described by a binary matrix of the interconnections  $\mathbf{A}_{pP}^b, \mathbf{A}_{pG}^b, \mathbf{F}_{pP}^b, \mathbf{F}_{pG}^b$ . Based on the matrix of interconnections concerning the respective design, the binary matrix  $\mathbf{Z}^b$  has been elaborated.

Table 9.4 includes the binary matrix  $\mathbf{Z}^b$  which shows the structure of the consumption of energy carriers in the set of designs in the analyzed building. Design No 2, for instance, comprising the production of electricity in a CHP system with a piston combustion engine (second column in Table 9.3) is connected with a consumption of electricity and diesel oil (row 1 and 14 in the matrix  $\mathbf{Z}^b$ ). In the case of the analyzed building, the supply of energy carriers is not connected with the consumption of other energy carriers; therefore column 1 in the matrix  $\mathbf{Z}^b$  contains only zero elements. Other external supplies (drinking water, natural gas, heating oil, and diesel oil) are delivered from outside.

The set of all possible variants of the energy management system have been formulated by choosing one design from each project. The number of variants which may be realized, determined in such a way, is considerable. For example, in the being considered case the number of all possible variants has been calculated as  $N = 396$ . However, many of those variants are not justified from the viewpoint described in Sect. 9.2. In the considered example, after the elimination of unnecessary solutions, a subset of 28 variants of the energy management system has been selected. For each variant of the energy management structure, the binary vector  $\mathbf{W}_l^b$  is found. Table 9.5 shows vectors  $\mathbf{W}_l^b$  for all the considered variants.

Each design is described by means of the binary matrices  $\mathbf{A}_{pP}^b$ ,  $\mathbf{A}_{pG}^b$ ,  $\mathbf{F}_{pP}^b$ ,  $\mathbf{F}_{pG}^b$  concerning the structure of the consumption and by-production of energy carriers.

On the basis of the matrices  $\mathbf{A}_{pP}^b$ ,  $\mathbf{A}_{pG}^b$ ,  $\mathbf{F}_{pP}^b$ ,  $\mathbf{F}_{pG}^b$  and Eqs. (9.5–9.13) the input–output matrices for all variants have been found. Table 9.6 shows an exemplary matrix  $\mathbf{A}_p^b + \mathbf{F}_p^b$  for the variant W4. An energy flow diagram of energy management of complex buildings can be drawn by using the matrix  $\mathbf{A}_p^b + \mathbf{F}_p^b$ .

Figure 9.6 shows the energy flow diagram of the variant W4 of the energy management structure. Variant W4 represents the application of a CHP unit powered with diesel oil in the analyzed office building. The CHP unit produces hot water for the absorption chiller besides heat and electricity. During the peak period heat is also produced in the peak water-heater fuelled with natural gas. The peak cold is delivered from the refrigeration chiller powered by electricity. The next step in the procedure is the investigation of the whole energy equipment in the energy management of complex buildings and the replacement of the binary elements of the matrix  $\mathbf{A}_p^b + \mathbf{F}_p^b$  with concrete coefficients of energy consumption. After replacing the binary elements, the mathematical model of the energy balance of complex buildings is applied [15]. Using this model, it is possible to calculate the consumption of all energy carriers concerning all the variants of the energy management structure.

### 9.7.3 Energy Balance for the Selected Variant of Energy Management of Complex Buildings

In a detailed energy analysis the variant W4 has been chosen (Fig. 9.6.). In the analysis of variant W4 the following data have been taken into account:

energy efficiency of peak gas boiler-  $\eta_{Eb} = 85\%$ ,  
 index of cogeneration of CHP unit (based on piston engine)-  $\sigma = 0.6$ ,  
 coefficient of the own needs (electricity) of the cogeneration unit-  $\varepsilon = 0.02$ ,  
 energy efficiency of the cogeneration unit-  $\eta_{E\text{CHP}} = 87.1\%$ ,  
 coefficient of performance of the absorption chiller-  $\text{COP}_{\text{abs}} = 0.7$   
 coefficient of performance of the peak compression refrigerator-  $\text{COP}_p = 3$ ,  
 energy efficiency of the system power plant fired with hard coal-  $\eta_{\text{el}} = 33.9\%$ .

Based on data concerning the project of the analyzed office building and characteristics of energy equipment, the annual consumption of the internal energy carriers for the variant W4 has been calculated. The results of the calculations have been presented below.

The annual electricity consumption in the particular energy branches is (according to Table 9.4):

$$Z_{11} = 54.91 \text{ GJ},$$



**Table 9.6** Binary Input–output matrix  $\mathbf{A}_p^b + \mathbf{F}_p^b$ —variant W4

Energy carrier/Project		1	2	3	4	5	6	7	8	9	10
1	Electricity	1	1	1	1	1	1	1	1	1	0
2	Heat	0	0	0	1	0	1	1	1	0	0
3	Cold	0	0	0	0	1	0	0	0	0	0
4	Hot process water for abs. chiller I	0	0	1	0	0	0	0	0	0	0
5	Cooling medium I 6/12 °C	0	0	0	0	0	0	0	0	1	0
6	Hot process water I 60/45 °C	0	0	0	0	0	0	0	0	0	0
7	Hot process water II 85/55 °C	0	0	0	0	0	0	0	0	1	0
8	Hot tap water	0	0	0	0	0	0	0	0	0	0
9	Air from air conditioning unit	0	0	0	0	0	0	0	0	0	0
10	Ventilation air	0	0	0	0	0	0	0	0	0	0
11	Drinking water	0	0	0	0	0	0	0	1	0	0
12	Natural gas	0	1	0	0	0	0	0	0	0	0
13	Heating oil	0	0	0	0	0	0	0	0	0	0
14	Diesel oil	1	1	0	0	0	0	0	0	0	0

$$Z_{12} = 22.05 \text{ GJ},$$

$$Z_{13} = 75.25 \text{ GJ},$$

$$Z_{14} = 11.03 \text{ GJ},$$

$$Z_{15} = 140.3 \text{ GJ},$$

$$Z_{16} = 7.88 \text{ GJ},$$

$$Z_{17} = 44.1 \text{ GJ},$$

$$Z_{18} = 3.62 \text{ GJ},$$

$$Z_{19} = 269.9 \text{ GJ}.$$

The annual consumption of heat for the production of hot process water for the absorption chiller is:

$$Z_{24} = 1,290.697 \text{ GJ}.$$

The annual consumption of heat for the production of hot process water 60/45 °C is:

$$Z_{26} = 1,677.11 \text{ GJ}.$$

The annual consumption of heat for the production of hot process water 85/55 °C is:

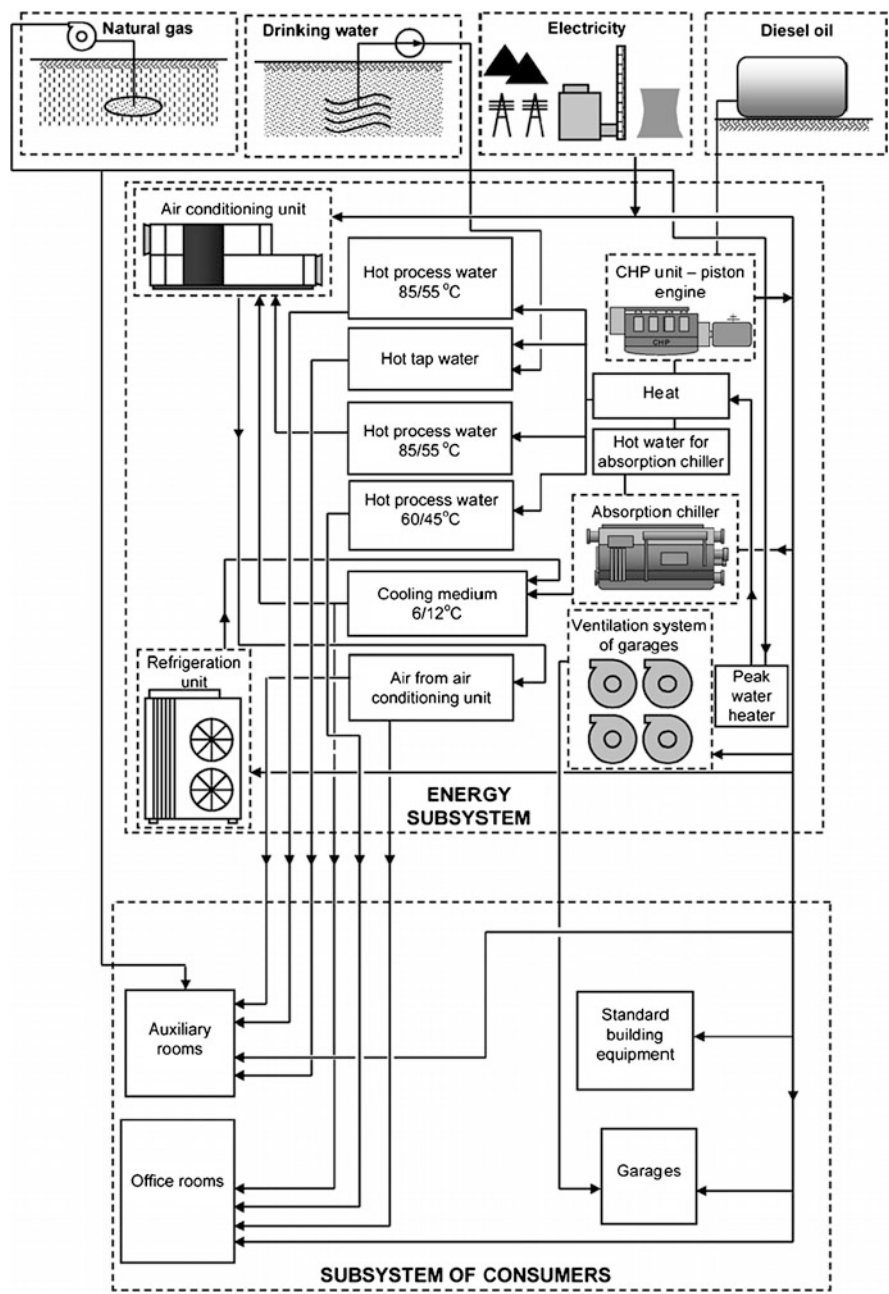


Fig. 9.6 Flow diagram of the energy management structure—variant W4

$$Z_{27} = 1,827.66 \text{ GJ.}$$

The annual consumption of heat for the production of hot tap water is:

$$Z_{28} = 170.53 \text{ GJ.}$$

The overall annual heat demand is:

$$Q = Z_{24} + Z_{26} + Z_{27} + Z_{28} = 4,965.33 \text{ GJ.}$$

The annual heat production by the CHP unit is:

$$Q_{\text{CHP}} = 3,972.26 \text{ GJ.}$$

The annual heat production by the peak gas boiler is:

$$Q_P = 993.07 \text{ GJ.}$$

The annual consumption of cold for the production of the cooling medium (water-glycol 6/12 °C) is:

$$Z_{35} = 1,129.36 \text{ GJ.}$$

The annual production of the cooling medium by the peak compression refrigerator is:

$$Q_{Cp} = 22,587 \text{ GJ.}$$

The annual production of the cooling medium by the absorption chiller is:

$$Q_{Cabs} = 903.49 \text{ GJ.}$$

The annual heat consumption by the absorption chiller is:

$$Z_{43} = 1,290.697 \text{ GJ.}$$

The annual consumption of the cooling agent by the air-conditioning unit is:

$$Z_{59} = 69.78 \text{ GJ.}$$

The annual heat consumption by the air-conditioning unit is:

$$Z_{79} = 975.9 \text{ GJ.}$$

The annual consumption of drinking water for the production of hot tap water is:

$$Z_{118} = 994.6 \text{ Mg.}$$

The annual consumption of the chemical energy of natural gas for the production of heat (in peak boilers) is:



$$Z_{122} = \frac{Q_p}{\eta_{Eb}} = 1,168.47 \text{ GJ} \quad (9.38)$$

In the considered variant W4 electricity is a by-product of the CHP unit. It has been assumed that heat is the main product (as in system CHP units). Heat produced by the CHP unit is consumed entirely by the analyzed complex building. The peak demand for heat is covered by the gas boiler. The annual by-production of electricity  $U_{12}$  is calculated from relation:

$$U_{12} = Q_{CHP} \cdot \sigma = 2,383.36 \text{ GJ}$$

The consumption of the chemical energy of diesel oil for the production of electricity by the CHP unit is calculated based on the principle of avoided expenditure of fuel (similar to the principle of avoided costs). According to this principle, the production of electricity in the CHP unit should be charged with the consumption of the chemical energy of fuel as in the replaced system power plant:

$$Z_{141} = \frac{U_{12}}{\eta_{Eel}} = 7,030.56 \text{ GJ} \quad (9.39)$$

The annual consumption of the chemical energy of diesel oil for heat production is:

$$Z_{142} = \frac{Q_{CHP} + U_{12}}{\eta_{E CHP}} - \frac{U_{12}}{\eta_{E el}} = 266.37 \text{ GJ} \quad (9.40)$$

PES (Primary Energy Savings) for this CHP unit is described by the relation:

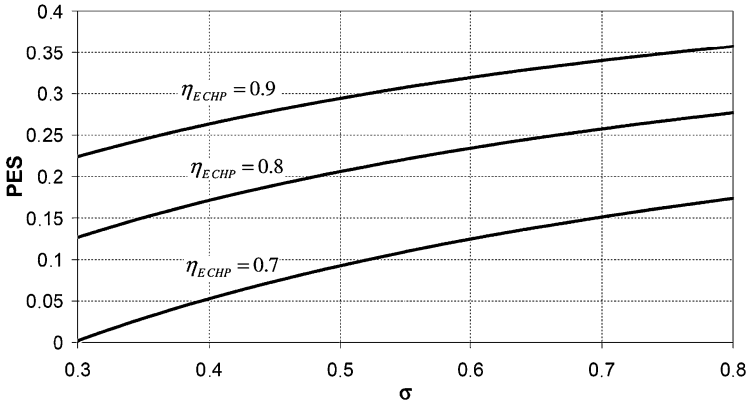
$$\text{PES} = 1 - \frac{1 + \frac{\sigma}{\eta_{E CHP}}}{\frac{1}{\eta_{ref h}} + \frac{\sigma}{\eta_{ref el}}} = 0.297 \quad (9.41)$$

where

$\eta_{ref h} = 0,9$  reference efficiency of heat production,  
 $\eta_{ref el} = 0,4$  reference efficiency of electricity production

PES calculated for the cogeneration unit under consideration represent a rather high value. Such a high value results from the high value of the considered energy efficiency of the cogeneration unit and its index of cogeneration. Figure 9.7 shows values of PES versus these coefficients ( $\sigma$  and  $\eta_{E CHP}$ ).

Table 9.7 presents the results of a systems analysis for the variant W4 of energy management of the analyzed office building. It includes the annual consumption of all energy carriers considered in the building. Figure 9.8 presents a comparison of two variants (W1 and W4) with respect to the annual consumption of energy carriers which are supplied to the office building. W1 is characterized by supplying heat and electricity to the building from outside.



**Fig. 9.7** PES versus to  $\sigma$  and  $\eta_{E_{CHP}}$

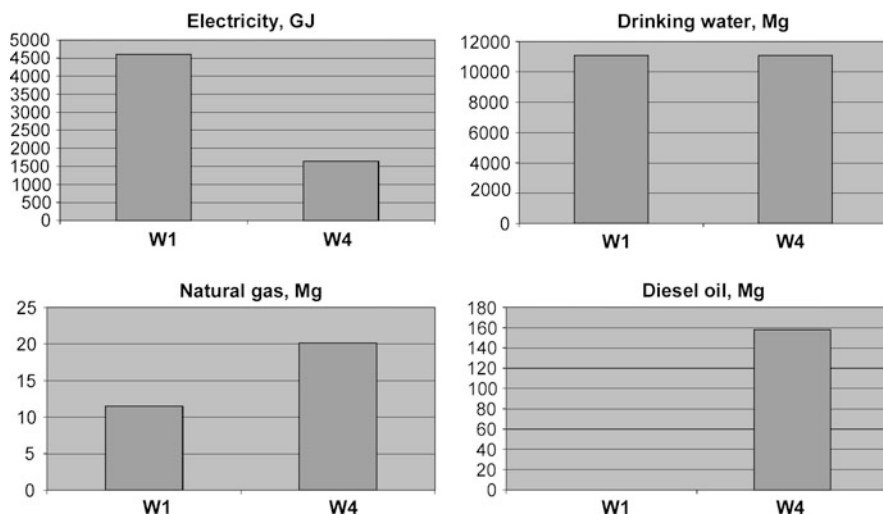
Due to the installation of the CHP the supply of electricity from outside has been reduced 3-fold, whereas the supply of natural gas has been increased 2-fold. For the variant with CHP the supply of diesel oil is indispensable.

The analysis has been performed based on the mathematical model of the energy balance of complex buildings [15]. The results of this analysis may be useful in further investigations in order to find the optimal variant of energy management of complex buildings.

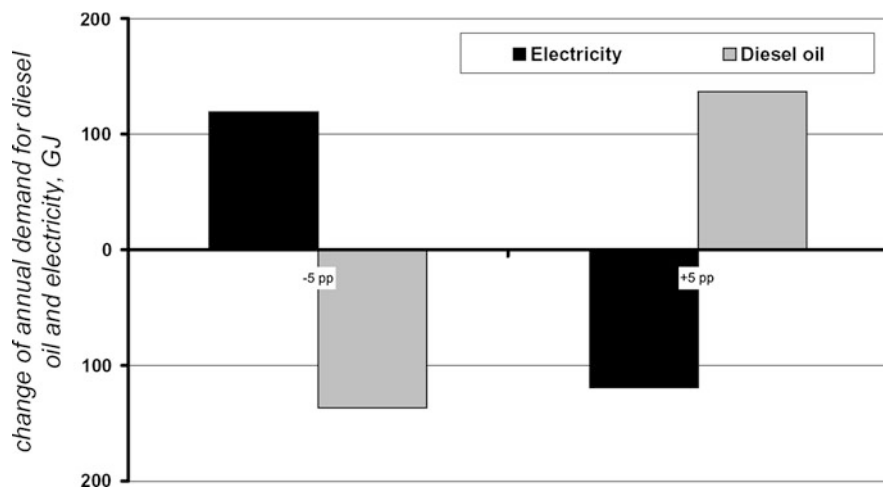
The uncertainty of the input data influences the reliability of the obtained results of calculation in a various degree. In order to assess this influence a sensitivity analysis has been applied. The data which exert the largest influence on the energy consumption in the variant W4 of the analyzed building have been chosen to analyse the sensitivity. The uncertainty of the following coefficients has been considered:  $\sigma$  -CHP coefficient,  $\eta_{E_{CHP}}$  -efficiency of the CHP unit, and  $COP_{abs}$  -coefficient of performance of the absorption chiller. Figures 9.9, 9.10, and 9.11 show exemplary results of the sensitivity analysis.

The increase of the cogeneration coefficient (a higher production of electricity with the same demand for heat) involves a reduction in the supply of electricity from outside. Although the fuel input in CHP increases simultaneously, the energy savings of fuel also increase in comparison with separate production of heat and electricity. The increase of CHP efficiency, which amounts to 3 percentage point (pp) effects a reduction of about 212 GJ of diesel oil per year. The increased coefficient of performance (COP) of the absorption chiller by 10 (pp) effects a drop in diesel oil consumption of about 171 GJ per year and the consumption of natural gas of about 27 GJ per year. Simultaneously, however, the supply of electricity increases due to the decreased production in cogeneration.

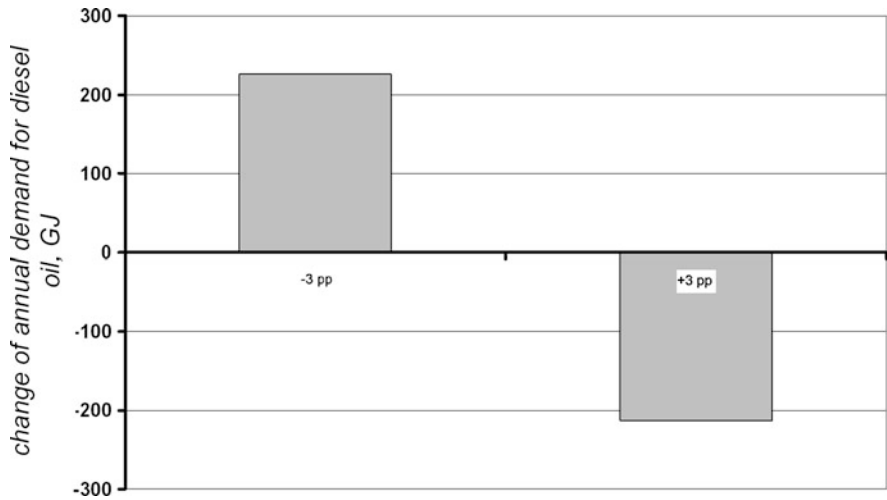




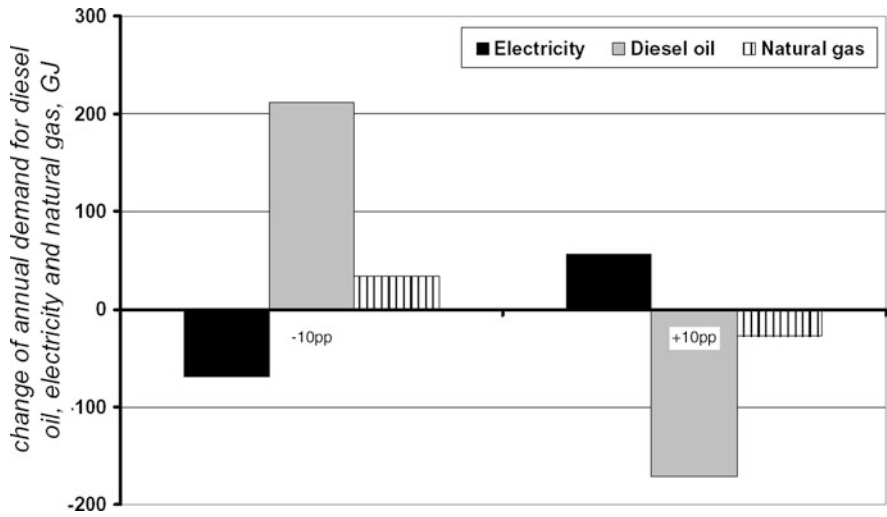
**Fig. 9.8** Annual consumption of energy carriers which are supplied to the analyzed office building—variant W1 and variant W4 of energy management structure



**Fig. 9.9** Change of annual demand for diesel oil and electricity supplied to the building caused by variation of CHP coefficient  $\sigma$



**Fig. 9.10** Change of annual demand for diesel oil (in GJ) supplied to the building with respect to the energy efficiency of the CHP unit  $\eta_{E_{CHP}}$



**Fig. 9.11** Change of annual demand for electricity, diesel oil, and natural gas supplied to the building caused by variation of coefficient of performance of absorption chiller  $COP_{abs}$

## 9.8 Example of the Application of an Optimization Model for the Choice of an Energy-Management Structure for Industrial-Urban Complex

### 9.8.1 Scenario of the Energy Management of the Industrial-Urban Complex

The subject of the preliminary design is the choice of the optimal structure of the energy subsystem producing energy carriers both for the needs of industrial and municipal consumers [22]. Such industrial-urban complexes are often encountered in industrial regions. The industrial subsystem is not only the consumer of energy carriers but also the producer of some of them (e.g., by-production of low-calorific gas or waste heat).

It has been decided that the energy management for the industrial-urban complex under consideration is to base on a CHP plant. The installation of a back-pressure turbo-generator or extraction condensing turbo-generator may be applied. The boiler-house of the CHP will be equipped with double-fuel boilers. These may be either gas and coal or gas and oil boilers. The fundamental fuel in the boilers will be low-calorific technological fuel gas provided by the subsystem of industrial consumers. Low-pressure steam will be used for space heating and ventilation, as well as for the production of hot tap water (mainly for the subsystem of municipal consumers) and also for technological purposes in the subsystem of industrial consumers. For the production of process air it is planned to install either blowers driven by steam turbines (turbo-blowers) or blowers driven by electric motors. The choice of equipment for the water-softening plant has not been analyzed, because its capacity is determined by the nominal capacity of the steam boilers.

The subsystem of consumers in this industrial-urban complex comprises industrial processes and municipal branches (e.g., a district heating system and a municipal water system). Table 9.8 contains the binary vector  $\mathbf{O}^b$  describing the structure of the demand for energy carriers by the subsystem of consumers. Nonzero elements provide information about energy carriers directly consumed in the subsystem of consumers. Zero elements concern energy carriers which are consumed only in the energy subsystem. The set of energy equipment and engines for the production of the considered energy carriers is also shown in Table 9.8.

Ten kinds of energy carriers obtained as main products have been distinguished, one energy carrier (low-calorific technological fuel gas) being a by-product which does not supplement the main production, as well as three energy carriers supplied entirely from outside.

Based on a general specification of energy carriers and specification of engines and energy equipment, presented in Table 9.8, a set of projects and designs has been established with nine projects and 12 designs distinguished (Table 9.9). The symbol  $U_4 \wedge U_5 \wedge D_4$ , for instance, denotes the back-pressure steam turbine and pressure-reducing valve, as well as the external supply of electrical energy ( $D_4$ ).

**Table 9.8** General specification of energy carriers, structure vector  $O^b$  of the demand for energy carriers for the technological subsystem and set of energy equipment and engines

Energy carrier	$O^b$	Equipment or engines	Symbol
Medium-pressure steam	0	Steam boilers fired with low-calorific gas and coal	$U_1$
		Steam boilers fired with low-calorific gas and oil	$U_2$
Low-pressure steam 1	1	Extraction-condensing turbine (steam bleeder	$U_3$
Electrical energy	1	0.8 MPa)	
		Back-pressure turbine (exhaust pressure 0.8 MPa)	$U_4$
		Pressure-reducing valve 3.7/0.8 MPa	$U_5$
Low-pressure steam 2	0	Pressure-reducing valve 0.8/0.12 MPa	$U_6$
Process air	1	Process air turbo-blowers	$U_7$
		Electrically driven process air blowers	$U_8$
Heat	1	Heat exchangers	$U_9$
Soft water	0	Water-softening plant	$U_{10}$
Feed water	0	Deaerating heater and pumping station of feed water	$U_{11}$
Industrial and drinking water	1	Pumping station of industrial and drinking water	$U_{12}$
Compressed air	1	Air compressors	$U_{13}$
Power coal	0		
Fuel oil	0		
Natural gas	1		
Low-calorific gas	1		

**Table 9.9** Projects and designs

t	Project	p	Design
1	Medium-pressure steam	1	$U_1$
		2	$U_2$
2	Low-pressure steam 1	3	$U_3 \wedge U_5 \wedge D_4$
	Electrical energy	4	$U_4 \wedge U_5 \wedge D_4$
3	Low-pressure steam 2	5	$U_6$
4	Process air	6	$U_7$
		7	$U_8$
5	Heat	8	$U_9$
6	Soft water	9	$U_{10}$
7	Boiler water	10	$U_{11}$
8	Industrial and drinking water	11	$U_{12}$
9	Compressed air	12	$U_{13}$

### 9.8.2 Elaboration of the Set of Variants Concerning the Energy Management System and Determination of the Structure of the Binary Input–Output Matrix [20]

According to this scenario (Tables 9.8 and 9.9) the set of all possible variants of the industrial-municipal energy system has been formulated, choosing one design from each project. In the example being considered eight variants of the energy

**Table 9.10** Variants of energy subsystems

Variant 1	Project—t								
	1	2	3	4	5	6	7	8	9
I	$U_1$	$U_3 \wedge U_5 \wedge D_4$	$U_6$	$U_7$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$	$U_{13}$
II	$U_1$	$U_3 \wedge U_5 \wedge D_4$	$U_6$	$U_8$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$	$U_{13}$
III	$U_1$	$U_4 \wedge U_5 \wedge D_4$	$U_6$	$U_7$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$	$U_{13}$
IV	$U_1$	$U_4 \wedge U_5 \wedge D_4$	$U_6$	$U_8$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$	$U_{13}$
V	$U_2$	$U_3 \wedge U_5 \wedge D_4$	$U_6$	$U_7$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$	$U_{13}$
VI	$U_2$	$U_3 \wedge U_5 \wedge D_4$	$U_6$	$U_8$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$	$U_{13}$
VII	$U_2$	$U_4 \wedge U_5 \wedge D_4$	$U_6$	$U_7$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$	$U_{13}$
VIII	$U_2$	$U_4 \wedge U_5 \wedge D_4$	$U_6$	$U_8$	$U_9$	$U_{10}$	$U_{11}$	$U_{12}$	$U_{13}$

system have been formed (Table 9.10). Figure 9.12 presents a schematic diagram of the energy subsystem of this industrial-urban complex for one of the eight variants, viz.,  $\{U_1, U_4 \wedge U_5 \wedge D_4, U_6, U_7, U_9, U_{10}, U_{11}, U_{12}, U_{13}\}$  -variant III.

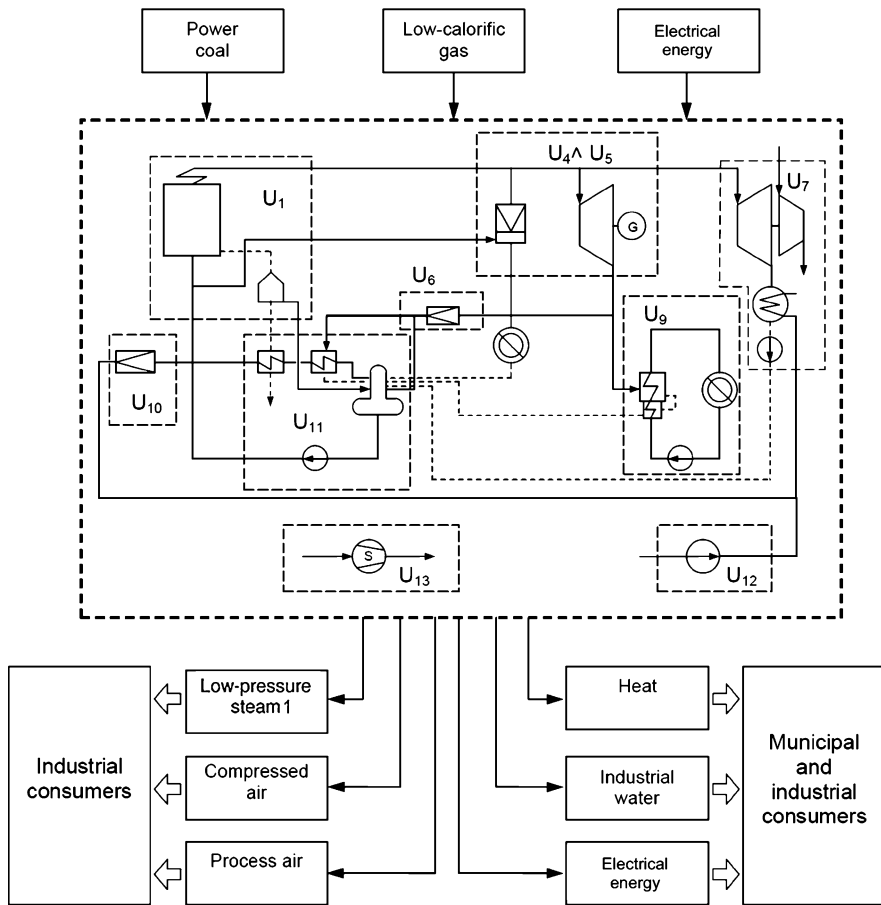
Each design is described by means of the binary matrices  $\mathbf{A}_{pP}^b, \mathbf{A}_{pG}^b, \mathbf{F}_{pP}^b, \mathbf{F}_{pG}^b$  concerning the structure of the consumption and by-production of energy carriers. For example, matrix  $\mathbf{A}_G^b$  for variant III is determined as follows:

$$\mathbf{A}_G^b = \mathbf{A}_{1G}^b + \mathbf{A}_{4G}^b + \mathbf{A}_{5G}^b + \mathbf{A}_{6G}^b + \mathbf{A}_{8G}^b + \mathbf{A}_{9G}^b + \mathbf{A}_{10G}^b + \mathbf{A}_{11G}^b + \mathbf{A}_{12G}^b \quad (9.42)$$

Matrix  $\mathbf{A}_{1G}^b$  (design  $U_1$  -steam boilers fired with low-calorific technological gas and coal) has a nonzero first column relating to the first column of the matrix  $\mathbf{A}_G^b$ . Matrix  $\mathbf{A}_{4G}^b$  (design  $U_4 \wedge U_5 \wedge D_4$  -back-pressure steam turbine and pressure-reducing valve 3.7/0.8 MPa) has a nonzero second column relating to the second column of the matrix  $\mathbf{A}_G^b$ . The nonzero elements of this column concern the consumption of medium-pressure steam and boiler water in the pressure-reducing valve 3.7/0.8 MPa. The coefficients concerning the back-pressure steam turbine are presented in the matrix  $\mathbf{A}_{4P}^b$ . Matrix  $\mathbf{A}_{5G}^b$  (design  $U_6$  -pressure-reducing valve 0.8/0.12 MPa) has a nonzero third column (the third column of matrix  $\mathbf{A}_G^b$ ). Matrix  $\mathbf{A}_{6G}^b$  (design  $U_7$  -air-process turbo-blowers) has a nonzero fifth column relating to the fifth column of the matrix  $\mathbf{A}_G^b$ . Matrix  $\mathbf{A}_{8G}^b$  (design  $U_9$  -heat exchangers) has a nonzero sixth column concerning the sixth column of the matrix  $\mathbf{A}_G^b$ . Matrix  $\mathbf{A}_{9G}^b$  (design  $U_{10}$  -water softening plant) has a nonzero seventh column (the seventh column of the matrix  $\mathbf{A}_G^b$ ). The matrices  $\mathbf{A}_{10G}^b, \mathbf{A}_{11G}^b, \mathbf{A}_{12G}^b$  (the designs  $U_{11}$  -de-aereating heater and pumping station for boiler water,  $U_{12}$  -pumping station for industrial water,  $U_{13}$  -air compressor) have nonzero columns numbered 8, 9, and 10, respectively. These nonzero columns relate to columns 8, 9, and 10 in the matrix  $\mathbf{A}_G^b$ . The other columns of matrix  $\mathbf{A}_G^b$  (4, 11, 12, and 13) are zero-columns. Table 9.11 presents the matrix  $\mathbf{A}_G^b$  for variant III.

The matrices  $\mathbf{A}_P^b, \mathbf{F}_P^b$  and  $\mathbf{F}_G^b$  for variant III are identical in size to matrix  $\mathbf{A}_G^b$ . Matrix  $\mathbf{A}_P^b$  has only two nonzero elements:  $a_{12}^P$  and  $a_{92}^P$ , concerning the





**Fig. 9.12** Schematic diagram of energy management of the industrial-urban complex

consumption of medium pressure steam and industrial water by the back-pressure steam turbine. Matrix  $\mathbf{F}_P^b$  has nonzero element- $U_1$  concerning the by-production of electrical energy in the CHP plant. Matrix  $\mathbf{F}_G^b$  has three nonzero elements  $f_{31}^G, f_{75}^G, f_{76}^G$  related to the by-production of low-pressure steam 2 (expander after the blow-down of the boiler), the by-production of soft water (condensate) in the turbo-blowers and heat exchangers, respectively.

### 9.8.3 Structural Analysis of the Binary Input–Output Matrix

Table 9.12 presents the binary input–output matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  obtained by means of summing up the matrices  $\mathbf{A}_P^b$ ,  $\mathbf{A}_G^b$ ,  $\mathbf{F}_P^b$ ,  $\mathbf{F}_G^b$  making use of the principles of

**Table 9.11** “Input–output” binary matrix  $\mathbf{A}_G$  (variant III)

Energy carrier	$i \setminus j$	1	2	3	4	5	6	7	8	9	10	11	12	13
Medium-pressure steam	1	0	1	0	0	1	0	0	0	0	0	0	0	0
Low-pressure steam 1	2	0	0	1	0	1	1	0	0	0	0	0	0	0
Low-pressure steam 2	3	0	0	0	0	0	0	0	1	0	0	0	0	0
Electrical energy	4	1	0	0	0	1	1	1	1	1	1	0	0	0
Process air	5	0	0	0	0	0	0	0	0	0	0	0	0	0
Heat	6	0	0	0	0	0	0	0	0	0	0	0	0	0
Soft water	7	0	0	0	0	0	0	0	1	0	0	0	0	0
Boiler water	8	1	1	0	0	0	0	0	0	0	0	0	0	0
Industrial water	9	0	0	0	0	1	0	1	0	0	1	0	0	0
Compressed air	10	0	0	0	0	0	0	0	0	0	0	0	0	0
Power coal	11	1	0	0	0	0	0	0	0	0	0	0	0	0
Natural gas	12	0	0	0	0	0	0	0	0	0	0	0	0	0
Low-calorific gas	13	1	0	0	0	0	0	0	0	0	0	0	0	0

**Table 9.12** Input–output matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  variant III

Energy carrier	$i \setminus j$	1	2	3	4	5	6	7	8	9	10	11	12	13
Medium-pressure steam	1	0	1	0	0	1	0	0	0	0	0	0	0	0
Low-pressure steam 1	2	0	0	1	0	1	1	0	0	0	0	0	0	0
Low-pressure steam 2	3	1	0	0	0	0	0	0	1	0	0	0	0	0
Electrical energy	4	1	1	0	0	1	1	1	1	1	1	0	0	0
Process air	5	0	0	0	0	0	0	0	0	0	0	0	0	0
Heat	6	0	0	0	0	0	0	0	0	0	0	0	0	0
Soft water	7	0	0	0	0	1	1	0	1	0	0	0	0	0
Boiler water	8	1	1	0	0	0	0	0	0	0	0	0	0	0
Industrial water	9	0	1	0	0	1	0	1	0	0	1	0	0	0
Compressed air	10	0	0	0	0	0	0	0	0	0	0	0	0	0
Power coal	11	1	0	0	0	0	0	0	0	0	0	0	0	0
Natural gas	12	0	0	0	0	0	0	0	0	0	0	0	0	0
Low-calorific gas	13	1	0	0	0	0	0	0	0	0	0	0	0	0

Boolean algebra. In this matrix, formed in the order of a universal specification, there are 12 nonzero elements below the main diagonal, which render it difficult to solve the set of balance equations.

Next the matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  is transformed by means of the algorithm of structural analysis. First of all the energy branches are divided into three groups: “input”-type, “centre”-type, and “output”-type. The division of the matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  into three groups is shown in Table 9.13. In the input–output binary matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  divided into blocks, the number of nonzero elements below the main diagonal has been reduced to only five.

In the next step, strongly coherent subsystems are determined among the energy branches belonging to the center group. The center matrix  $(\mathbf{A}^b + \mathbf{F}^b)_c$  is then

**Table 9.13** Input–output binary matrix  $(\mathbf{A}^b + \mathbf{F}^b)$  divided into blocks

Energy carrier	$i \setminus j$	1	2	3	4	5	6	7	8	9	10	11	12	13
Electrical energy	1	0	0	0	1	1	0	1	1	1	1	1	1	0
Power coal	2	0	0	0	1	0	0	0	0	0	0	0	0	0
Low-calorific gas	3	0	0	0	1	0	0	0	0	0	0	0	0	0
Medium-pressure steam	4	0	0	0	0	1	0	0	0	0	1	0	0	0
Low-pressure steam 1	5	0	0	0	0	0	1	0	0	0	1	1	0	0
Low-pressure steam 2	6	0	0	0	1	0	0	0	1	0	0	0	0	0
Soft water	7	0	0	0	0	0	0	0	1	0	1	1	0	0
Boiler water	8	0	0	0	1	1	0	0	0	0	0	0	0	0
Industrial water	9	0	0	0	0	1	0	1	0	0	1	0	1	0
Process air	10	0	0	0	0	0	0	0	0	0	0	0	0	0
Heat	11	0	0	0	0	0	0	0	0	0	0	0	0	0
Compressed air	12	0	0	0	0	0	0	0	0	0	0	0	0	0
Natural gas	13	0	0	0	0	0	0	0	0	0	0	0	0	0

**Table 9.14** Set of the successive powers of the matrix  $(\mathbf{A}^b + \mathbf{F}^b)_c$ 

$(\mathbf{A}^b + \mathbf{F}^b)_c$	$(\mathbf{A}^b + \mathbf{F}^b)_c^2$
$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$
$(\mathbf{A}^b + \mathbf{F}^b)_c^3$	$(\mathbf{A}^b + \mathbf{F}^b)_c^4$
$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$

transformed. For this purpose the matrix  $\mathbf{C}$  is calculated from Eq. (9.14). In Table 9.14 the successive powers of the matrix  $(\mathbf{A}^b + \mathbf{F}^b)_c$  have been gathered.

The successive matrices  $\mathbf{C}$  according to Eqs. (9.14) and (9.15) are as follows:

$$\mathbf{C}_1 = \sum_{s=1}^1 (\mathbf{A}^b + \mathbf{F}^b)_c^s = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C}_2 = \sum_{s=1}^2 (\mathbf{A}^b + \mathbf{F}^b)_c^s = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{C}_3 = \sum_{s=1}^3 (\mathbf{A}^b + \mathbf{F}^b)_c^s = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{C}_4 = \sum_{s=1}^4 (\mathbf{A}^b + \mathbf{F}^b)_c^s = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Because:

$$\mathbf{C}_3 = \mathbf{C}_4$$

we can write:

$$\mathbf{C} = \mathbf{C}_3 = \mathbf{C}_4$$

The matrix intersection  $\mathbf{W}$  is deduced from the following equation:

$$\mathbf{W} = \mathbf{C} \cap \mathbf{C}^T$$

Matrix  $\mathbf{W}$  has nonzero elements only in those places, where matrix  $\mathbf{C}$  has nonzero elements as well as matrix  $\mathbf{C}^T$ . In this case matrix  $\mathbf{W}$  takes the following form [10]:

$$\mathbf{W} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \text{-medium-pressure steam} \\ \text{-low-pressure steam 1} \\ \text{-low-pressure steam 2} \\ \text{-soft water} \\ \text{-boiler water} \\ \text{-industrial water} \end{array}$$

**Table 9.15** Input–output binary matrix ( $\mathbf{A}^b + \mathbf{F}^b$ ) transformed into a block-triangular matrix with a minimal number of feedback elements

Energy carrier	$i \setminus j$	1	2	3	4	5	6	7	8	9	10	11	12	13
Electrical energy	1	0	0	0	1	1	0	1	1	1	1	1	1	0
Power coal	2	0	0	0	0	0	0	0	1	0	0	0	0	0
Low-calorific gas	3	0	0	0	0	0	0	0	1	0	0	0	0	0
Industrial water	4	0	0	0	0	1	0	0	0	0	1	0	1	0
Soft water	5	0	0	0	0	0	0	1	0	0	1	1	0	0
Low-pressure steam 2	6	0	0	0	0	0	0	1	1	0	0	0	0	0
Boiler water	7	0	0	0	0	0	0	0	1	1	0	0	0	0
Medium-pressure steam	8	0	0	0	0	0	0	0	0	1	1	0	0	0
Low-pressure steam 1	9	0	0	0	0	0	1	0	0	0	1	1	0	0
Process air	10	0	0	0	0	0	0	0	0	0	0	0	0	0
Heat	11	0	0	0	0	0	0	0	0	0	0	0	0	0
Compressed air	12	0	0	0	0	0	0	0	0	0	0	0	0	0
Natural gas	13	0	0	0	0	0	0	0	0	0	0	0	0	0

In the analyzed example the intersection matrix  $\mathbf{W}$  contains one strongly coherent subsystem involving the following energy carriers: medium-pressure steam, low-pressure steam 1, low-pressure steam 2, and boiler water. The other two branches (industrial water and soft water) do not compose a strongly coherent subsystem due to the lack of feedback connections. They correspond to zero-rows and zero-columns in the intersection matrix  $\mathbf{W}$ .

The order of rows and columns in the matrix  $\mathbf{W}$  is the same as in Table 9.13. Rearranging the fourth and fifth row, as well as the fourth and fifth column, we obtain the matrix  $\bar{\mathbf{W}}$  as a matrix with diagonally-arranged blocks:

$$\bar{\mathbf{W}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The final form of the matrix  $(\mathbf{A}^b + \mathbf{F}^b)$ , after topological classification, is presented in Table 9.15. Below the main diagonal there is only one nonzero element corresponding to the coefficient of the consumption of low-pressure steam 1, used for the production of low-pressure steam 2. According to the arrangement of the rows in the topologically arranged matrix  $\mathbf{A}_G^b$  first the balances concerning the energy branches belonging to the output block are set up, beginning with natural gas. Next, the branches of the center block are balanced taking into account the iterative loop concerning the low-pressure steam 1 (feedback). The calculations are completed by the balance of electricity winding up the “input” block. As already mentioned, the balancing of the energy carriers, the determination of the coefficients of consumption  $a_{ij}$ , and by-production  $f_{ij}$ , as well as the choice of

energy equipment, are interdependent tasks. Therefore, the presented sequence of balancing is simultaneously the sequence of choosing the energy equipment in the respective energy branches.

### 9.8.4 Input Data

Figures 9.13, 9.14, and 9.15 present the duration curves for the following energy carriers

- duration curves of the sale of low-pressure steam 1-  $\dot{K}_9(\tau)$ -Fig. 9.13,
- duration curves of the global demand for heat-  $\dot{Q}_{11}(\tau)$ -Fig. 9.14,
- duration curves of the global demand for compressed air-  $\dot{G}_{12}(\tau)$ -Fig. 9.15.

Tables 9.16 and 9.17 present the first approximation of the unit costs of energy carriers, as well as the technical coefficients of the consumption, and by-production of energy carriers for a strongly coherent subsystem.

The coefficients  $a_{89}$  and  $a_{79}$  are calculated from the following equations:

$$a_{89} = \frac{a_{89}^P P_9 + a_{89}^G G_9}{P_9 + G_9}, \quad (9.43)$$

$$a_{79} = \frac{a_{79}^G G_9}{P_9 + G_9} \quad (9.44)$$

where

$a_{89}^P = 1$	coefficient of the consumption of medium-pressure steam for the basic part of the production of low-pressure steam 1,
$a_{89}^G$	coefficient of the medium-pressure steam consumption by the pressure-reducing valve 3.7/0.8 MPa,
$P_9, G_9$	basic and variable (peak) part of low-pressure steam 1 production,
$a_{79}^G$	coefficient of boiler water consumption by the pressure-reducing valve 3.7/0.8 MPa

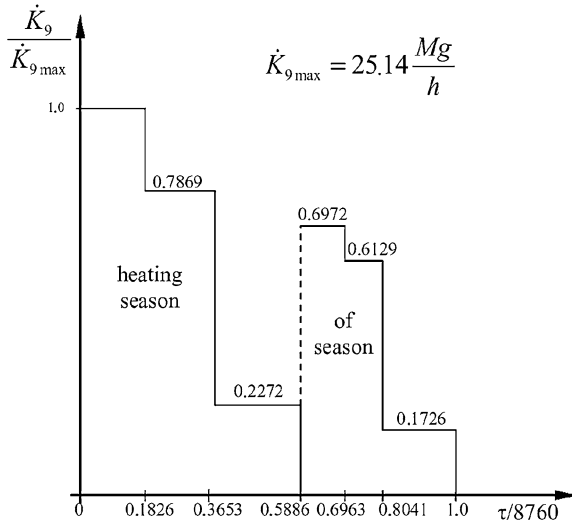
The coefficients  $a_{89}^G$  and  $a_{79}^G$  are calculated from the mass and energy balance of the pressure-reducing valve 3.7/0.8 MPa (Fig. 9.12):

$$a_{89}^G = \frac{h_9 - h_7}{h_8 - h_7}, \quad (9.45)$$

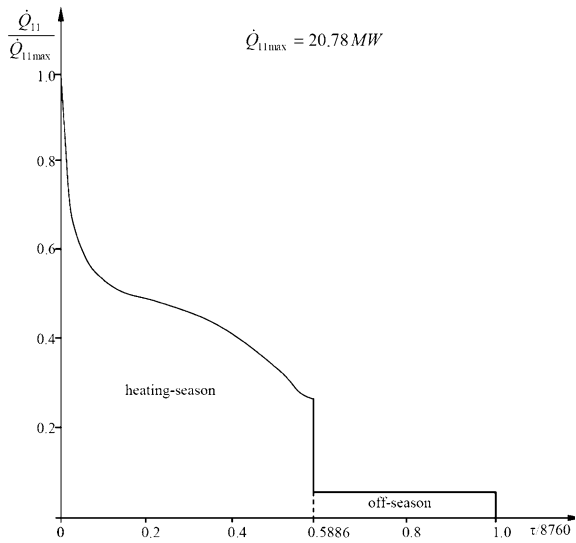
$$a_{79}^G = \frac{h_8 - h_9}{h_8 - h_7} \quad (9.46)$$

where:  $h_7$ ,  $h_8$ , and  $h_9$  denote the specific enthalpy of boiler water, medium-, and low-pressure steam 1, respectively.

**Fig. 9.13** Duration curve concerning the sale of low-pressure steam 1



**Fig. 9.14** Duration curve of the global demand for heat

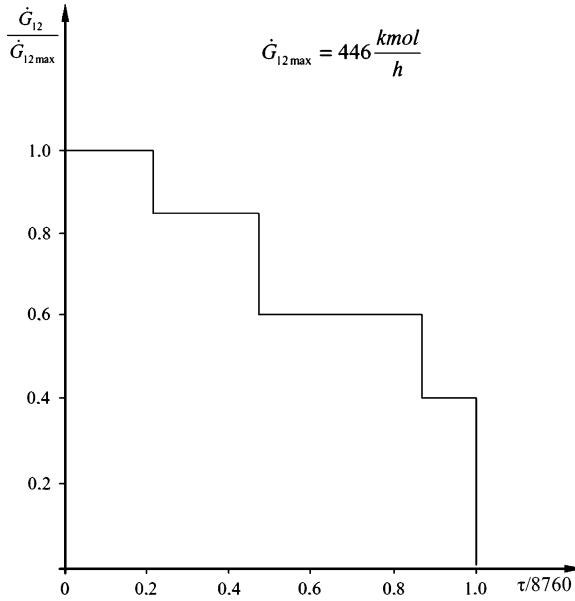


The coefficient  $f_{68}^G$  of the by-production of low-pressure steam 2 is calculated from the mass and energy balance of the expander after the blowdown of the boiler (Fig 9.12)

$$f_{68}^G = \zeta \frac{h'_8 - h'_6}{h''_6 - h'_6} \quad (9.47)$$

where

**Fig. 9.15** Duration curve concerning the global demand for compressed air



**Table 9.16** Unit costs of energy carriers—first approximation

i	Energy carrier	Unit	Average cost
1	Electrical energy	m.u. */MWh	1,850.0
2	Power coal	m.u./GJ	76.5
3	Low-calorific gas	m.u./GJ	76.5
4	Industrial water	m.u./10 <sup>3</sup> Mg	580.0
5	Soft water	m.u./Mg	85.0
6	Low-pressure steam 2	m.u./Mg	238.0
7	Boiler water	m.u./Mg	93.0
8	Medium-pressure steam	m.u./Mg	265.0
9	Low-pressure steam 1	m.u./Mg	252.0
10	Process air	m.u./Mmol	2,915.0
11	Heat	m.u./GJ	106.0
12	Compressed air	m.u./Mmol	2,466.0
13	Natural gas	m.u./GJ	97.6

\* m.u.- monetary unit

- $\zeta$  ratio of amount of saline water for the blow-off to the production of medium-pressure steam,
- $h'_8, h'_6$  specific enthalpy of saline water from the blow-off and thickened saline water from the expander, respectively,
- $h''_6$  specific enthalpy of saturated vapor from the expander (low-pressure steam 2).



**Table 9.17** Coefficients of consumption and by-production of energy carriers for a strongly coherent subsystem—first approximation

Energy carrier		Unit	Value	
			heating-season	off-season
Medium-pressure steam	$a_{89}$	Mg/Mg	0.9847	1.0000
Low-pressure steam 1	$a_{96}$	Mg/Mg	1.0000	1.0000
Low-pressure steam 2	$f_{68}$	Mg/Mg	0.0138	0.0138
	$a_{67}$	Mg/Mg	0.0875	0.0800
Boiler water	$a_{78}$	Mg/Mg	1.0600	1.0600
	$a_{79}$	Mg/Mg	0.0153	0.0000

Based on the mass and energy balances of the deaerated water (Fig 9.12) the coefficients  $a_{67}^G$  and  $a_{57}^G$  describing the consumption of soft water and low-pressure steam 2 for the production of boiler water, we obtain [16]:

$$a_{67}^G = \frac{1}{h_6 - h_5} \left[ h_7 - h_5 + \frac{G_{cT}}{G_7} (h_5 - h_{cT}) + \frac{G_{10}}{G_7} f_{5\ 10}^G (h_5 - h_{cB}) + \frac{G_{11}}{G_7} f_{5\ 11}^G (h_5 - h_{cH}) \right] \quad (9.48)$$

$$a_{57}^G = 1 - a_{67}^G - \frac{1}{G_7} (G_{cT} + f_{5\ 10}^G G_{10} + f_{5\ 11}^G G_{11}) \quad (9.49)$$

where

$h_5, h_6, h_7$ , specific enthalpy of soft water, low-pressure steam 2, and boiler water, respectively,  
 $G_{cT}$  amount of condensate from the subsystem of industrial consumers,  
 $G_7, G_{10}, G_{11}$  production of boiler water, process air and heat, respectively,  
 $f_{5\ 10}^G, f_{5\ 11}^G$  coefficients of the by-production of condensate from turbo-blowers and heat exchangers, respectively,  
 $h_{cT}, h_{cB}, h_{cH}$  specific enthalpy of the condensate from the subsystem of industrial consumers, process air turbo-blowers, and heat exchangers, respectively.

### 9.8.5 Algorithms for the Determination of the Optimal Power Rating and Nominal Capacity of the Engines and Energy Equipment (variant III) [19]

The order of energy carriers in Table 9.15 is final and obligatory in further calculations. The optimal power rating and capacity of the engines and energy equipment have been determined for each variant by means of Lagrange's method of decomposition. Individual optimization algorithms for each energy carrier are employed.

### A. Compressed air

The nominal capacities of air compressors and their number are determined by means of the comparison of variants. The following assumptions have been made:

- (a) the air compressors have the same nominal capacity,
- (b) the maximum capacity of the air compressor assembly in each variant must exceed the maximum demand of compressed air for the considered subsystem of consumers,
- (c) additionally, one air compressor must be foreseen as a reserve.

The objective function for the comparison of the variants in this case takes the following form:

$$\Phi_{12} = (\rho_{G12} + \beta_{G12})I_{G12} + \alpha_{G12}\dot{G}_{n12} + (\varepsilon_{G12} + a_{1\ 12}^G k_1 + a_{4\ 12}^G k_4)G_{12} \rightarrow \min \quad (9.50)$$

where

$\dot{G}_{n12}$	nominal capacity of the air compressor assembly,
$a_{1\ 12}^G, a_{4\ 12}^G$	coefficients of the consumption of electrical energy and industrial water, respectively,
$k_1, k_4$	average unit cost of electrical energy and industrial water,
$G_{12}$	annual production of compressed air

Two variants are considered. The following data are identical for both variants: pressure ratio—8.8;  $\rho_{G12} = 0.11$ ;  $\beta_{G12} = 0.029$ ;  $\varepsilon_{G12} = 291$  m.u.\*/Mmol. The remaining data are presented in Table 9.18.

The results of the comparison of the considered variants are presented in Table 9.18. Variant 1 has been chosen for the design. A similar approach was applied in the case of the branch process air.

### B. Low-pressure steam 1

Low-pressure steam 1 from the back-pressure steam turbine is the basic part of the production. The peak part of low-pressure steam production is obtained from the pressure-reducing valve 3.7/0.8 MPa. The objective function for calculating the optimal power rating of the back-pressure steam turbine has the following form:

$$\begin{aligned} \Phi_9 = & (\rho_{P9} + \beta_{P9})I_{P9} + (\rho_{G9} + \beta_{G9})I_{G9} + \alpha_{P9}\dot{P}_{n9} + \alpha_{G9}\dot{G}_{n9} \\ & + (\varepsilon_{P9} + a_{49}^P k_4 + a_{89}^P k_8 - f_{19}^P k_1)P_9 + (\varepsilon_{G9} + a_{79}^G k_7 + a_{89}^G k_8)G_9 \rightarrow \min \end{aligned} \quad (9.51)$$

where

$I_{P9}, I_{G9}$	investments outlay concerning the back-pressure turbine and pressure-reducing valve station,
------------------	--

**Table 9.18** Data and calculation results for the determination of the air-compressor assembly

Quantity	Unit	V a r i a n t	
		1	2
Nominal capacity	kmol/h	446	281
Power of electric motor	kW	1,250	750
Number of compressors		2	3
Capital expenditure	mln m.u. <sup>a</sup>	88.4	92.09
$\alpha_{G12}\dot{G}_{n12}$	mln m.u./year	3.1	4.35
$a_{112}^G$	MWh/Mmol	2.6502	2.4821
$a_{412}^G$	10 <sup>3</sup> Mg/Mmol	0.3475	0.3116
$\Phi_{12}$ (final iteration)	mln m.u./year	30.9	31.6

<sup>a</sup> m.u. monetary unit

$\dot{P}_{n9}, \dot{G}_{n9}$	flux of low-pressure steam 1 from the back pressure steam turbine with nominal load and nominal capacity of the pressure-reducing valve 3.7/0.8 MPa, respectively,
$P_9, G_9$	annual basic and peak part of low-pressure steam 1 production, respectively,
$a_{49}^P, a_{89}^P$	coefficients of the consumption of industrial water and medium-pressure steam for the back-pressure steam turbine,
$a_{79}^G, a_{89}^G$	coefficients of the consumption of boiler water and medium-pressure steam for the pressure-reducing valve station,
$f_{19}^P$	coefficient of the by-production of electric energy in the back-pressure steam turbine,
$k_1, k_4, k_7, k_8$	unit costs of electric energy, industrial water, boiler water, and medium-pressure steam, respectively.

The algorithm for the determination of the power rating for the back-pressure steam turbine was formulated based on the following assumptions:

- all the electricity produced in the CHP plant is consumed by the considered subsystem of consumers; moreover, the domestic electrical power system supplies the considered municipal-urban complex,
- the investment outlay for the steam-boiler house of the CHP plant does not depend on the power rating of the back-pressure steam turbine,
- the cost of repairs and maintenance, prime cost and operating costs (without the cost of energy carriers) do not depend on the power rating of the back-pressure steam turbine,
- the efficiency of the steam boiler is independent of the power rating of the back-pressure steam turbine,
- the pressure-reducing valve 3.7/0.8 MPa should secure the peak part of the production of low-pressure steam 1, and cover the demand for low-pressure steam 1 in case the back-pressure steam turbine should fail,
- the influence of the power rating of the back-pressure steam turbine on the investment outlay of the pressure-reducing valve 3.7/0.8 MPa is inconsiderable,

- (g) the influence of the power rating of the back-pressure steam turbine on the costs of boiler water for the steam boilers and pressure-reducing valve 3.7/0.8 MPa, as well as the cost of industrial water for the back-pressure steam turbine are also inconsiderable.

The objective function now takes the form:

$$\begin{aligned}\Phi_9 = & (\rho_{P9} + \beta_{P9})I_{P9} + (\rho_{G9} + \beta_{G9})I_{G9} \\ & + \alpha_{P9}\dot{P}_{n9} + \alpha_{G9}\dot{G}_{n9} + \varepsilon_{P9}P_9 + \varepsilon_{G9}G_9 \\ & + (a_{49}k_4 + a_{79}k_7 + a_{89}k_8 - f_{19}k_1)\Omega_9 \rightarrow \min\end{aligned}\quad (9.52)$$

where

$$\Omega_9 = P_9 + G_9 \quad (9.53)$$

The values of the coefficients  $a_{79}$ ,  $a_{89}$  are determined by means of the Eqs. (9.44), (9.43). The coefficients  $a_{49}$  and  $a_{19}$  are expressed similarly. Due to the assumption (c), (f), and (g) the following term of the objective function is constant:

$$\begin{aligned}C_1 = & \beta_{P9}I_{P9} + (\rho_{G9} + \beta_{G9})I_{G9} + \alpha_{P9}\dot{P}_{n9} + \alpha_{G9}\dot{G}_{n9} \\ & + \varepsilon_{P9}P_9 + \varepsilon_{G9}G_9 + (a_{49}k_4 + a_{79}k_7)\Omega_9\end{aligned}\quad (9.54)$$

Based on the balance of costs concerning the medium-pressure steam we get:

$$k_{G8} = a_{18}k_1 + a_{28}k_2 + a_{38}k_3 + a_{78}k_7 - k_{F6}f_{68} + \frac{S_8}{\Omega_8} \quad (9.55)$$

where

$\Omega_8 = G_8$	annual production of medium-pressure steam,
$f_{68} = f_{68}^G$	coefficient of the by-production of low-pressure steam 2 Eq. (9.47),
$a_{18}, a_{28}, a_{38}, a_{78}$	coefficients of the consumption of electrical energy, power coal, low-calorific gas, and boiler water for the steam boilers,
$k_1, k_2, k_3, k_{F6}, k_7, k_{G8}$	-unit costs of electrical energy, power coal, low-calorific gas, by-production of low-pressure steam 2, boiler water, and medium-pressure steam, respectively,
$S_8$	-arbitrary fixed costs (without the costs of energy carriers) of the steam boilers.

Introducing the expression (9.55) into Eq. (9.52) we obtain:

$$\Phi_9 = \rho_{P9}I_{P9} + (a_{28}k_2 + a_{38}k_3)a_{89}\Omega_9 - k_{1f_{19}}\Omega_9 + C_2 \rightarrow \min \quad (9.56)$$

where

$$C_2 = \left( a_{18}k_1 + a_{78}k_7 - f_{68}k_{F6} + \frac{S_8}{\Omega_8} \right) a_{89}\Omega_9 + C_1. \quad (9.57)$$

The influence of the power rating of the back-pressure steam turbine on the term  $C_2$  may be neglected.

The annual costs of the by-production of electrical energy is given by:

$$k_1 f_{19} \Omega_9 = \frac{f_{19} \Omega_9}{\eta_{\text{Epp}} \eta_{\text{et}}} k_2 + K_{\text{rem}} \quad (9.58)$$

where

- $\eta_{\text{Epp}}$  energy efficiency of the production of electrical energy in the reference system electrical power station fired with hard coal,
- $\eta_{\text{et}}$  efficiency of the transmission of electrical energy,
- $k_2$  unit cost of power coal,
- $K_{\text{rem}}$  remaining costs (without the cost of hard coal) of the production of electricity in the replaced electrical power station.

The term  $K_{\text{rem}}$  has no influence on the optimal of power rating of the back-pressure steam turbine.

The following expressions have been introduced which denote consumption of the chemical energy of fuels in the boilers and the production of electricity:

$$(a_{28} + a_{38}) a_{89} \Omega_9 = \frac{a_{89} \Omega_9 \Delta i_9}{\eta_{\text{Eb}}} + E_{\text{ch el}} \quad (9.59)$$

$$f_{19} \Omega_9 = E_{\text{el}} \quad (9.60)$$

where

- $\Delta i_9$  decrease of specific enthalpy of low-pressure steam 1 supplied to consumers,
- $\eta_{\text{Eb}}$  energy efficiency of steam boilers,
- $E_{\text{ch el}}$  annual consumption of the chemical energy of fuels in steam boilers charging the production of electricity,
- $E_{\text{el}}$  annual production of electricity in the back-pressure turbo-generator.

Introducing Eqs. (9.58) to (9.60) into (9.56) and assuming that  $k_2 = k_3$  (low calorific gas replaces hard coal with the same efficiency of boiler), we obtain:

$$\Phi_9 = \rho_{p9} I_{p9} + \left( E_{\text{ch el}} - \frac{E_{\text{el}}}{\eta_{\text{Epp}} \eta_{\text{et}}} \right) k_2 + C \rightarrow \min \quad (9.61)$$

where

$$C = C_2 + \frac{a_{89} \Omega_9 \Delta i_9}{\eta_{\text{Eb}}} k_2 + K_{\text{rem}} \quad (9.62)$$

The influence of the power rating of the back-pressure steam turbine on the term  $C$  may be neglected.

The annual production of electrical energy  $E_{el}$  is determined from the equation:

$$E_{el} = N_{el\ n} \tau_n + \int_{\tau_n}^{\tau_a} N_{el} d\tau \quad (9.63)$$

where:  $N_{el\ n}, N_{el}$  - power rating and instantaneous value of the load of the back-pressure turbo-generator,  $\tau_n, \tau_a$  - annual duration of operation of the turbo-generator with power rating load and the annual action duration of the turbo-generator.

The annual consumption of the chemical energy of fuels in the steam boilers for the production of electrical energy is given by:

$$E_{ch\ el} = \frac{1}{\eta_{Eb}} \left[ \frac{N_{el\ n}}{\eta_{me\ n}} \tau_n + \int_{\tau_n}^{\tau_a} \frac{N_{el}}{\eta_{me}} d\tau \right] \quad (9.64)$$

where  $\eta_{me\ n}, \eta_{me}$  denotes the electromechanical efficiency of the turbo-generator with power rating load, as well as with instantaneous load.

The investment outlay characteristic for back-pressure turbine is given by the following equation:

$$I_{p9} = a_T c_T c_n N_{el\ n}^{0.7} \quad (9.65)$$

where

- $a_T$  coefficient dependent on the thermodynamic parameter of incoming steam,
- $c_T$  unit capital expenditure for the back-pressure steam turbine, m.u./MW,
- $c_n$  coefficient dependent on the number of turbo-generators.

Figure 9.16 presents the duration curve concerning the global demand for low-pressure steam 1. The duration curve is divided into two parts: heating season and off-season.

The power rating of the back-pressure steam turbine is calculated from the following equation:

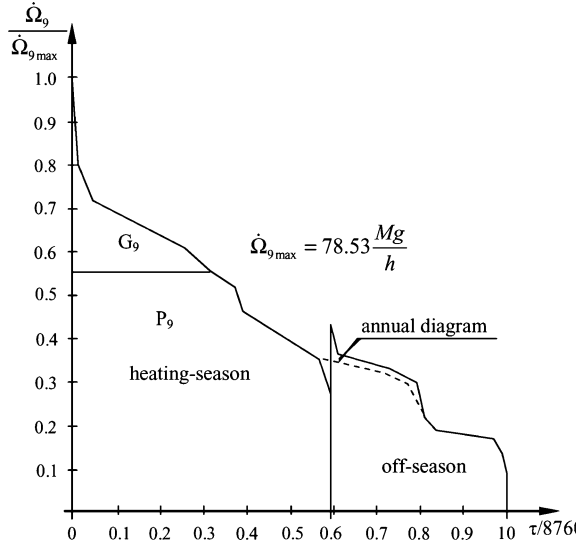
$$N_{el\ n} = \dot{P}_{9\ max} \Delta h_T \eta_{in} \eta_{me\ n} \quad (9.66)$$

where

- $\dot{P}_{9\ max}$  maximum flux of exhaust steam from the back-pressure steam turbine,
- $\Delta h_T$  isentropic decrease of specific enthalpy in the back-pressure steam turbine,
- $\eta_{in}$  nominal isentropic efficiency of the back-pressure steam turbine.

The efficiencies  $\eta_{in}$  and  $\eta_{me\ n}$  are calculated from the following empirical [12] equations:

**Fig. 9.16** Duration curve concerning the global demand for low-pressure steam 1



$$\eta_{in} = -0.1531N_{eln}^{-0.914} + 0.853 \quad (9.67)$$

$$\eta_{me n} = 0.0126N_{eln} + 0.856 \quad (9.68)$$

Based on the energy characteristics of the back-pressure steam turbine [16], the electrical power  $N_{el}$  is determined:

$$N_{el} = \frac{\dot{P}_9 - Xd_n N_{eln}}{(1 - X)d_n} \quad (9.69)$$

and

$$d_n = \frac{1}{\Delta h_T \eta_{in} \eta_{me n}} \quad (9.70)$$

where

$\dot{P}_9$  flux of exhaust steam from the back-pressure steam turbine,

$X$  coefficient of idle run of the turbo-generator.

The coefficient  $X$  and electromechanical efficiency  $\eta_{me}$  are calculated from the following equations [12]:

$$X = 0.0041N_{eln}^2 - 0.0465N_{eln} + 0.342 \quad (9.71)$$

$$\eta_{me} = \eta_{me n} \left[ 1 - 0.646 \left( 1 - \frac{N_{el}}{N_{eln}} \right)^2 + 1.745 \left( 1 - \frac{N_{el}}{N_{eln}} \right)^4 - 2.099 \left( 1 - \frac{N_{el}}{N_{eln}} \right)^6 \right] \quad (9.72)$$

The optimal power rating of the back-pressure steam turbine is determined based on the condition:

$$\varphi = \rho_{p9} a_T c_T c_n N_{el n}^{0.7} + k_2 \left\{ \frac{1}{\eta_{Eb}} \left[ \frac{N_{el n}}{\eta_{me n}} \tau_n + \int_{\tau_n}^{\tau_a} \frac{N_{el}}{\eta_{me}} d\tau \right] - \frac{1}{\eta_{Epp} \eta_{et}} [N_{el n} \tau_n + \int_{\tau_n}^{\tau_a} N_{el} d\tau] \right\} \rightarrow \min \quad (9.73)$$

Equation (9.73), as well as the auxiliary expressions (9.67)–(9.72) constitute the algorithm for the determination of the optimal power rating of the back-pressure steam turbine. The duration curve concerning the global demand for low-pressure steam 1 (Fig. 9.16) is the fundamental set of values. Other data are:  $\eta_{Epp} = 0.34$ ;  $\eta_{et} = 0.96$ ;  $k_2 = 76.5 \text{ m.u./GJ}$ ;  $\eta_{Eb} = 0.78$ ;  $a_T = 1$ ;  $c_T = 25 \text{ m.u./MW}$ ;  $c_n = 1$ ;  $\rho_T = 0.11$ ; thermodynamic parameters of medium pressure-steam:  $p = 3 \text{ MPa}$ ,  $t = 435^\circ \text{C}$ ; pressure of exhaust steam from the back-pressure steam turbine  $-0.8 \text{ MPa}$ ; and temperature of condensate from the process and heat exchangers  $-80^\circ \text{C}$ .

It has been calculated, that this optimal power rating of the back-pressure steam turbine is  $N_{el n \text{ opt}} = 3.45 \text{ MW}$ . Figure 9.16 shows the field, covered by low-pressure steam 1 from the back-pressure steam turbine ( $P_9$ ), as well as the field covered by steam from the reducing-pressure valve  $3.7/0.8 \text{ MPa}$  ( $G_9$ ). The coefficient  $f_{19}^P$  of the by-production of electrical energy is calculated from the following equation:

$$f_{19}^P = \frac{1}{P_9} (N_{el n \text{ opt}} \tau_{n \text{ opt}} + \int_{\tau_{n \text{ opt}}}^{\tau_a} N_{el} d\tau) \quad (9.74)$$

The coefficient  $a_{49}^P$  of the consumption of industrial water was determined on the basis of catalogue data for a back-pressure steam turbine.

### C. Medium-pressure steam

Variant III of this energy management system foresees the installation of steam boilers fired with low-calorific gas (fundamental fuel) and hard coal.

The choice of steam boilers is made by comparing the following variants:

- (1) three steam boilers with a maximum capacity of 30 Mg/h,
- (2) three steam boilers with a maximum capacity of 35 Mg/h,
- (3) three steam boilers with a maximum capacity of 40 Mg/h,
- (4) four steam boilers with a maximum capacity of 30 Mg/h,

The following assumptions have been made:

- (a) the prime cost depends on the number of steam boilers,
- (b) the maximum fraction of chemical energy of power coal in the fuel mixture is 50 %,
- (c) the repair of steam boilers is foreseen in the off-season, and the idle time for repair concerns only one steam boiler.



The objective function has the following form:

$$\Phi_8 = (\rho_{G8} + \beta_{G8})I_{G8} + \alpha_{G8}\dot{G}_{n8} + (\varepsilon_{G8} + a_{18}^G k_1 + a_{28}^G k_2 + a_{38}^G k_3 + a_{78}^G k_7 - f_{68}^G k_{F6}) \cdot G_8 + K_{T8} \rightarrow \min \quad (9.75)$$

where

$\dot{G}_{n8}$	nominal capacity of the steam boilers,
$G_8$	annual production of the steam boilers,
$a_{18}^G, a_{28}^G, a_{38}^G, a_{78}^G$	coefficients of the consumption of electrical energy, power coal, low-calorific gas, and boiler water, respectively,
$f_{68}^G$	coefficient of the by-production of low-pressure steam 2,
$k_1, k_2, k_3, k_7$	average unit costs of electrical energy, power coal, low-calorific gas, and boiler water, respectively,
$k_{F6}$	unit cost of the by-production of low-pressure steam 2,
$K_{T8}$	annual costs of losses by the consumers due to the deficiency of medium-pressure steam.

The costs of losses due to failures of the boilers are presented in Table 9.19. In the case of the failure of a steam boiler first of all the turbo-blowers are turned off, and blowers with electrical drive are turned on.

The following data are used in the calculations:

- investment outlay characteristics of steam boilers:

$$I_{G8} = a_b c_b (0.25 n_8^{-0.3} + 0.75) n_8 \dot{G}_{n8}^{0.65} \quad (9.76)$$

where

$a_b$	-coefficient dependent on the thermodynamic parameters of steam; $a_k = 1$ for $p = 3.7$ MPa and $t = 450$ °C,
$c_b$	unit capital expenditure for the steam boiler; ( $c_k = 45$ mln m.u./Mg)
$n_8$	number of steam boilers,
$\dot{G}_{n8}$	nominal capacity of boiler.

- energy efficiency characteristics of steam boiler:

$$\eta_{Eb} = 0.307 z_g^2 - 0.361 z_g + 0.848 \quad (9.77)$$

where  $z_g$  denotes the share of chemical energy of low-calorific gas in the fuel mixture,

- maximum capacity characteristics of the steam boiler

$$\dot{G}_{8 \max} = \dot{G}_{8 \max o} (1 - 0.4 z_g) \quad (9.78)$$

where  $\dot{G}_{8 \max o}$  denotes the maximum capacity in the case of a coal-fired steam boiler.

**Table 9.19** Data and calculation results concerning the nominal capacity of steam boilers

Quantity	Unit	V a r i a n t			
		1	2	3	4
Nominal capacity	Mg/h	30	35	40	30
Number of steam boilers		3	3	3	4
$(\rho_{G8} + \beta_{G8})I_{G8}$	mln m.u. <sup>a</sup> /year	193.1	213.4	232.7	256.3
$\alpha_{G8}\dot{G}_{n8}$	mln m.u./year	15.7	15.7	15.7	17.1
$K_{T8}$	mln m.u./year	75.4	58.2	39.7	19.2
$a_{28}^G$	GJ/Mg	0.3117	0.1442	0.0751	0.0751
$a_{38}^G$	GJ/Mg	3.3699	3.4772	3.6221	3.6221
$\Phi_8$ (final iteration)	mln m.u./year	568.4	564.7	558.2	563.8

<sup>a</sup> m.u. monetary unit

In order to determine the coefficients  $a_{28}^G$  and  $a_{38}^G$ , concerning the consumption of hard coal and low-calorific gas the following data must be known:

- duration curve  $G_8(\tau)$  of the demand for medium-pressure steam (Fig. 9.17),
- energy characteristics of a steam boiler,
- duration curve of the chemical energy of the fuel mixture  $\dot{E}_{ch}(\tau)$ .

Using Eq. (9.77) the energy characteristic of a steam boiler has the following form:

$$\dot{E}_{ch} = \frac{\dot{G}_8 \Delta h_8}{0.307z_g^2 - 0.361z_g + 0.848} \quad (9.79)$$

where

- $\dot{E}_{ch}$  the flux of chemical energy of the fuel mixture,
- $\dot{G}_8$  flux of the production of medium-pressure steam,
- $\Delta h_8$  increase of specific enthalpy of steam in the boiler.

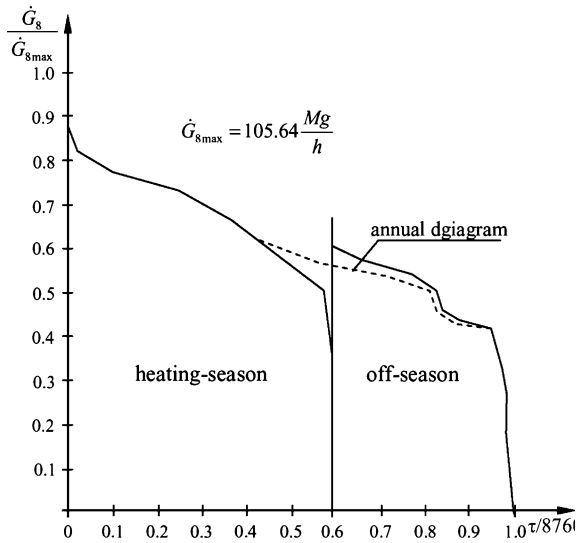
Introducing Eq. (9.78) into the energy characteristics (9.79) we obtain a right-side limitation of this characteristic:

$$\dot{E}_{ch \max} = \frac{\dot{G}_{8 \max} (1 - 0.4z_g) \Delta h_8}{0.307z_g^2 - 0.361z_g + 0.848}. \quad (9.80)$$

Surplus amounts of low-calorific gas are passed into double fuel boilers. The forecast demands for fuel for double-fuel boilers can be analyzed based on forecasts of the duration curves of the surplus of low-calorific fuel gas and the demand for live steam, as well as the energy characteristics of a double fuel boiler.

Figure 9.18 presents the method of determining the demand for chemical energy  $E_{chg}$  of low-calorific gas, as well as the chemical energy  $E_{chc}$  of hard coal for the steam boilers. The coefficients  $a_{28}$  and  $a_{38}$  are calculated from the equations:

**Fig. 9.17** Duration curve concerning the global demand for medium-pressure steam



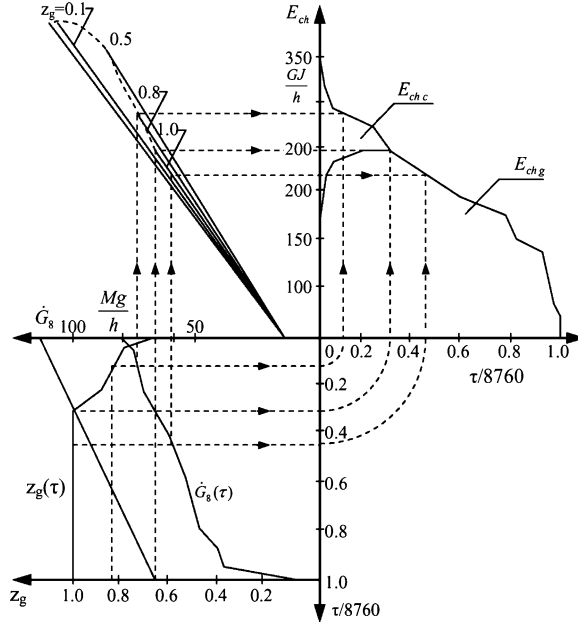
$$a_{28} = \frac{E_{chc}}{G_8} \quad (9.81)$$

$$a_{38} = \frac{E_{chg}}{G_8} \quad (9.82)$$

Figure 9.18 presents a full example of the forecasting analysis of the management of fuels in double-fuel boilers [24]. The bottom left-hand corner in Fig. 9.18 contains the duration curve  $\dot{G}_8(\tau)$  of the demand for live steam and the dependence on the maximum capacity of the boilers on the share of the chemical energy of low-calorific gas in the total chemical energy of fuels. Based on both these diagrams the diagram of the changing share of the chemical energy of low-calorific gas in the chemical energy of fuels was set up corresponding to the changes in the demand for steam in compliance with the duration curve  $\dot{G}_8(\tau)$ . In the upper left-hand corner the energy characteristics of a double-fuel boiler house are shown. The characteristics comprise a set of segments of straight lines concerning various shares  $z_g$  of low-calorific gas, which is a characteristic parameter. On the characteristics a broken line has been plotted limiting the family of straight lines on the right-hand side.

Based on the duration curve  $\dot{G}_8(\tau)$  expressing the demand for live steam and on the energy characteristics of double-fuel boilers the duration curve of the consumption of the chemical energy of fuel mix has been plotted. This duration curve is to be seen in the upper right-hand corner of Fig. 9.18, where also the curve  $E_{chg}$  describing the consumption of the surplus low-calorific gas, is also shown. The deficiency of the chemical energy of low-calorific gas must be compensated by combustion of an additional amount of hard coal  $E_{chc}$ .

**Fig. 9.18** Method of determining the demand for chemical energy of low-calorific gas and the chemical energy of power coal for the steam boilers [24];  $E_{ch\ g}$  chemical energy of low-calorific gas;  $E_{ch\ c}$  chemical energy of hard coal



The values of the coefficients  $a_{18}^G$ ,  $a_{58}^G$  and  $f_{68}^G$  have been determined based on information obtained by practical experience concerning the exploitation of double-fuel steam boilers.

The nominal capacity of steam boilers has been determined by comparing four variants. The results of the comparison of the considered variants are presented in Table 9.19. Variant three has been chosen for design.

#### D. Boiler water

The nominal capacities of boiler water pumps and their number are determined by means of the comparison of variants. The maximum capacity  $\dot{G}_{7\max}$  of the pumping station for boiler water should satisfy the following inequality:

$$\dot{G}_{7\max} \geq a_{78}^G \dot{G}_{8\max} + a_{79} \dot{\Omega}_{9\max} \quad (9.83)$$

where

$\dot{G}_{8\max}$ ,  $\dot{\Omega}_{9\max}$  maximum flux of medium-pressure steam and low-pressure steam 1, respectively,  
 $a_{78}^G$ ,  $a_{79}$  coefficients of the consumption of boiler water for steam boilers and the pressure-reducing valve 3.7/0.8 MPa.

In this example:

$$\dot{G}_{7\max} \geq 136.15 Mg/h$$

The objective function has the following form:

$$\Phi_7 = (\rho_{G7} + \beta_{G7})I_{G7} + \alpha_{G7}\dot{G}_{n7} + (\varepsilon_{G7} + a_{17}^G k_1)G_7 \rightarrow \min \quad (9.84)$$

where

- $\dot{G}_{n7}$  nominal capacity of the pumping station for boiler water,
- $G_7$  annual production of boiler water,
- $a_{17}^G$  coefficient of the consumption of electrical energy.

The following assumptions have been made:

- (a) all the boiler water pumps have the same nominal capacity,
- (b) the capacity of a boiler water pumping station should not reduce the capacity of the steam boiler,
- (c) additionally, one boiler water pump must be foreseen as a stand-by pump and two additional pumps driven by diesel-engines are foreseen,
- (d) the operating costs without the cost of electrical energy are the same for all the variants.

Three variants have been considered. The following data are identical for these variants:

$$\rho_{G7} = 0.114$$

$$\beta_{G7} = 0.029$$

Based on the duration curve of the demand for boiler water (Fig. 9.19) the considered variants have been analyzed. The energy characteristics of the boiler water pump assembly have been taken into account. The results of the comparison of the variants are presented in Table 9.20. Variant 2 has been chosen for design.

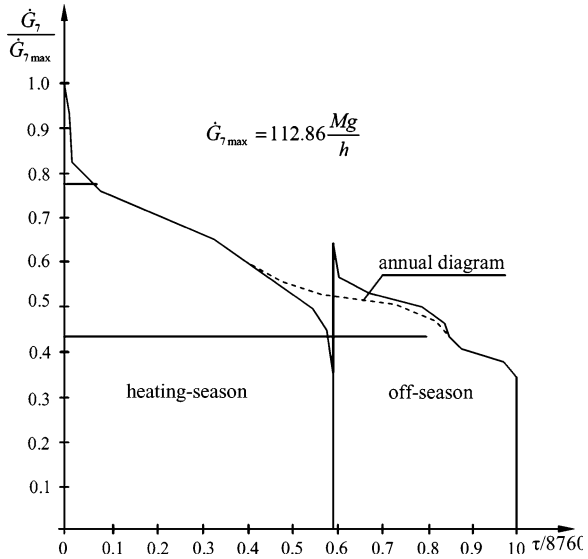
#### **D. Electrical energy**

Electrical energy is used to cover the needs of the subsystem of consumers of the industrial-urban complex and the internal needs of the energy subsystem. The duration curve of the global demand for electrical energy from the domestic electrical power system is presented in Fig. 9.20. This diagram has been set up taking into account the by-production of electrical energy by the back-pressure steam turbine.

### **9.8.6 Calculation Procedure and Results [16, 22]**

Calculations concerning the choice of the optimal structure of the energy management of the industrial-urban complex are an iterative procedure resulting from the applied decomposition of the global problem of optimization by means of Lagrange's method. The requirement of the compliance of the global criterion with local criteria (objective functions for particular energy carriers) is satisfied by the

**Fig. 9.19** Duration curve of the global demand for boiler water



procedure of coordination, that is, the matrix method of determining the unit costs of energy carriers.

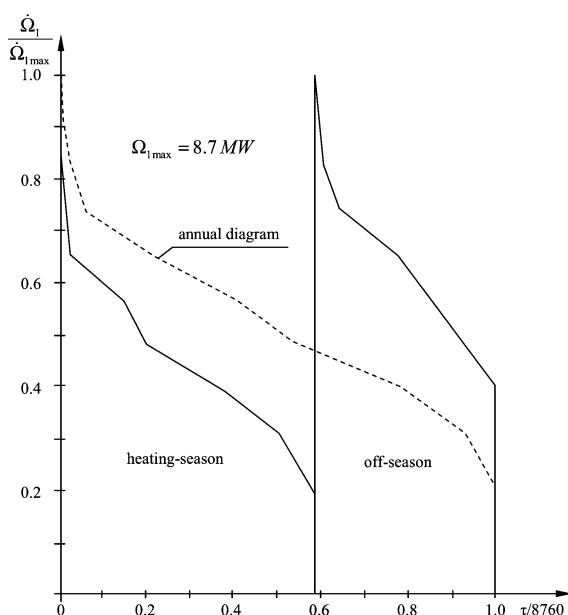
The coefficients of the consumption and by-production of energy carriers (final iteration) are collated in Table 9.21. Based on data provided in catalogues of energy equipment issued by the producers, five coefficients have been assumed, viz.  $a_{15}, a_{18}, a_{10}$  of the consumption of electrical energy when soft water, medium pressure steam and process air are being produced, as well as  $a_{49}$  and  $a_{412}$  which express the consumption of industrial water when low-pressure steam 1 and compressed air are being produced. The values of the coefficients  $a_{17}$  and  $a_{112}$  of the consumption of electrical energy driving the boiler-water pumps and air compressors, as well as the coefficient  $a_{810}$  of the consumption of medium-pressure steam driving the blowers were determined based on the characteristics of the power equipment and the duration curves of the demand for energy carriers. The coefficients  $a_{28}$  and  $a_{38}$  were determined by means of a forecasting analysis of the consumption of fuel in steam boilers. The values of the coefficients  $a_{57}$  and  $a_{67}$  of the consumption of soft water and low-pressure steam 2 are determined based on the deaereating heater balance. From the balance of the pressure-reducing valves, the values of the coefficients of the consumption of medium-pressure steam and boiler water are derived, whereas from the balance of the expander after the blowdown of the boiler, the coefficient  $f_{68}$  of the by-production of low-pressure steam 2 in the steam boilers is derived. The coefficient  $f_{19}^p$  of the by-production of electrical energy by a back-pressure steam turbine is determined by means of individual algorithms. The values of the coefficients  $a_{47}, a_{79}, a_{89}$  and  $f_{19}$  are determined after the division of the production of steam by the back-pressure turbine and the pressure-reducing valve 3.7/0.8 MPa. The remaining technical

**Table 9.20** Data and calculation results of the nominal capacity of boiler-water pumps

Quantity	Unit	V a r i a n t		
		1	2	3
Nominal capacity	Mg/h	27	45	75
Number of pumps		5	3	2
Driving power of electric motor	MW	0.08	0.16	0.20
Capital expenditure	mln m.u. <sup>a</sup>	10.9	8.1	7.0
$\alpha_{G7} \dot{G}_{n7}$	mln m.u./year	4.0	2.7	2.7
$a_{17}^G$	MWh/10 <sup>3</sup> Mg	0.00312	0.00276	0.00294
$\Phi_7$ (final iteration)	mln m.u./year	8.83	6.71	6.75

<sup>a</sup> m.u. monetary unit

**Fig. 9.20** Duration curve concerning the global demand for electrical energy from the national electrical power system



coefficients were evaluated based on the practical experience obtained from the existing energy subsystems (e.g., industrial water and heat).

The power rating and capacity of the engines and energy equipment have been determined each time in the iterative loop of the coordination process (the outer iterative loop of the investigated variant). In the case of energy branches constituting a strongly coherent subsystem it was necessary to select the equipment in the iterative loop comprising the determination of the values of the coefficients of consumption and by-production of energy carriers (interior iterative loop). In the example in question, four iterations had to be made within the process of determining the unit costs of energy carriers (Table 9.22). In each outer iterative loop, three iterations were accomplished in the interior loop of the strongly coherent

**Table 9.21** Technical coefficients of the consumption and by-production of energy carriers in the energy subsystem (final iteration)

i	Energy carrier	Consumer branch	Symbol	Unit	Value of coefficient
1	Electric energy	Boilers	$a_{18}$	MWh/Mg	0.0031
		Turbo-generator	$a_{19}$	MWh/Mg	0.0785
		Turbo-blowers	$a_{1\ 10}$	MWh/ Mmol	0.0157
		Heat exchangers	$a_{1\ 11}$	MWh/GJ	0.0017
		Water softening plant	$a_{15}$	MWh/Mg	0.0010
		Pumping station for boiler water	$a_{17}$	MWh/Mg	0.0028
		Pumping station for industrial water	$a_{14}$	MWh/ $10^3$ Mg	0.0900
		Air compressors	$a_{1\ 12}$	MWh/ Mmol	2.6502
2	Power coal	Steam boilers	$a_{28}$	GJ/t	0.0751
3	Low-calorific gas	Steam boilers	$a_{38}$	GJ/t	3.6221
4	Industrial and drinking water	Turbo-generator	$a_{49}$	$10^3$ Mg/ Mg	0.0007
		Turbo-blowers	$a_{4\ 10}$	$10^3$ Mg/ Mmol	0.4971
		Water softening plant	$a_{45}$	$10^3$ Mg/ Mg	0.0010
		Air compressors	$a_{4\ 12}$	$10^3$ Mg/ Mmol	0.3475
5	Soft water	Turbo-blowers	$f_5\ 10$	$10^3$ Mg/ Mmol	4.8842
		Heat exchangers	$f_5\ 11$	Mg/GJ	0.4000
		Pumping station of boiler water	$a_{57}$	Mg/Mg	0.8921
6	Low-pressure steam 2	Steam boilers	$f_{68}$	Mg/Mg	0.0138
		Pumping station of boiler water	$a_{67}$	Mg/Mg	0.1979
7	Boiler water	Steam boilers	$a_{78}$	Mg/Mg	1.0600
		Turbo-generator	$a_{79}$	Mg/Mg	0.0080
8	Medium-pressure steam	Turbo-generator and pressure-reducing valve 3.7/0.8 MPa	$a_{89}$	Mg/Mg	0.9920
		Turbo-blowers	$a_{8\ 10}$	Mg/Mmol	4.9839
9	Low-pressure steam 1	Pressure-reducing valve 0.8/0.12 MPa	$a_{96}$	MJ/Mg	1.0000
		Turbo-blowers	$a_{9\ 10}$	Mg/Mmol	0.4590
		Heat exchangers	$a_{9\ 11}$	Mg/GJ	0.4131

subsystem. Calculations in the interior iterative loop are brought to an end as soon as the required accuracy in the determination of the coefficients of consumption and by-production of energy carriers has been achieved. In the quoted example an



**Table 9.22** Ratio of unit coasts in the last and first iteration

t	Energy carrier	Ratio of unit costs in the last and first iteration
1	Electrical energy	1
2	Power coal	1
3	Low-calorific gas	1
4	Industrial water	1
5	Soft water	1.36
6	Low-pressure steam 2	2.96
7	Boiler water	2.26
8	Medium-pressure steam	2.89
9	Low-pressure steam 1	2.76
10	Process air	1.81
11	Heat	2.59
12	Compressed air	3.81
13	Natural gas	1

**Table 9.23** Power rating or nominal capacity, annual production, and external supplies

t	Energy carrier	Power rating or nominal capacity	Annual production and external supplies
1	Electrical energy	3,45 MW	65,285 <sup>a</sup> MWh
2	Power coal		39,600 GJ
3	Low-calorific gas		4,819 TJ
4	Industrial water	6,000 Mg/h	47,800 Gg
5	Soft water	40 Mg/h	231,900 Mg
6	Low-pressure steam 2	15 Mg/h	53,300 Mg
7	Boiler water	135 Mg/h	561,300 Mg
8	Medium-pressure steam	120 Mg/h	527,150 Mg
9	Low-pressure steam 1	35 Mg/h	328,050 <sup>b</sup> Mg
10	Process air	6,960 kmol/h	40,300 Mmol
11	Heat		180,000 GJ
12	Compressed air	892 kmol/h	2,940 Mmol
13	Natural gas		376,300 GJ

<sup>a</sup> own production–25,800 MWh<sup>b</sup> basic part of the production (back-pressure turbine-306,000 Mg)

accuracy of  $\varepsilon = 0.005$  has been assumed. The value  $\varepsilon$  corresponds to the accuracy in the solution of the balance equations of energy carriers amounting to about 0.3 %.

Calculations in the outer iterative loop are completed when the required accuracy in the determination of the unit cost of energy carriers has been reached. In this example it was assumed that  $\nu = 0.005$ . A better accuracy of calculations does not affect the choice of the optimal power and capacity of engines and energy equipment.

As a result of calculations of the choice of the optimal power rating and number of energy equipment concerning the variant presented in Fig. 9.12, the optimal set of engines and energy equipment could be determined (Table 9.23). The boiler house was equipped with three steam boilers with a nominal capacity of 40 Mg/h each, and is fired with low-calorific gas and hard coal. A back-pressure steam turbine was selected with a power rating of 3.45 MW. The peak production of low-pressure steam 1 is achieved in a set of three pressure-reducing valves 3.7/0.8 MPa with a nominal capacity of 35 Mg/h each. Low-pressure steam 2 is produced in a pressure-reducing valve 0.8/0.12 MPa with a capacity of 15 Mg/h. Two axial-flow blowers were selected with a capacity of 3.480 kmol/h, driven by condensation turbines with a power rating of 4.25 MW each. In the designed air-compressor assembly the installation of two radial compressors with a nominal capacity of 446 kmol/h has been chosen. In the boiler-water pumping station four pumps are installed with a capacity of 45 Mg/h each (one of them is to be a stand-by pump).

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